Article

Mathematical Philosophy of Consciousness

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Abstract

We propose a mathematical model of consciousness and information and use it to formulate empirical and theoretical approaches to both. Starting with previously introduced definition of information as a vector of concepts we extend it to a matrix. The diagonal elements of the matrix are identical to the original concepts of the vector and represent pure reactions, while the off-diagonal elements are correlators of concepts and represent associations. While transformation of information within a particular nervous system occurs due to numerous internal (biological, psychological, behavioral, individual history, and etc.) factors we nevertheless can consider all these factors as a combined force acting on information and causing its change. Thus we define consciousness as a matrix operator acting on matrix of information. Under adiabatic approximation we derive a Schrodinger-type equation governing dynamics of consciousness and information. In equilibrium (calm, normal) state all new information falls into expected range. When deviation from equilibrium state exceeds some critical value the system becomes unstable (alert, concerned) and resolves the instability by creating a new concept, which is a phase transition. We also suggest that feelings correspond to a partial loss of consciousness and mathematically represented by off-equilibrium values of correlators of concepts.

Keywords: Consciousness, mathematical philosophy, information, matrix, model, operator, Shrodinger equation, dynamics, equilibrium, correlation.

1. Introduction: Concepts and where information comes from

As it was recently realized (Rusakov 2022, 2023) the fundamental functional components of human consciousness are concepts. Likewise, the information that we seemingly "receive" may not exist (or be "received") without concepts of such information. The concepts are the notions we use in everyday life and are all familiar with. In order to perceive any information one has to have a concept of such information and of its components if there are ones. For example if you received such seemingly simple message as someone's date of birth, it does not guarantee that you received any information unless you possess a concept of such information in principle, as well as many other concepts such as a concept of birth, a concept of calendar (and thus concepts of numbers, years, months, days), and hundreds of other concepts, including language in which the message was delivered to you. If you do not have them, or do not understand the language, the message is just pure noise to you, bearing no information whatsoever.

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We often use notion of concept when referring to a scientific concept, but it requires some effort to realize that concepts are not only about science, they are everything we operate with. Such simple notions as "mother" or "father", "sit" or "stand", "tree" or "bush", "I" or "you", are concepts too. They are abstractions existing only in our consciousness. Therefore, in order to "receive" any information one needs to have a concept of such information. I put "receive" in quotation marks because we do not "receive" information, we *produce* it. What we receive is a signal, a message containing encoded data. These data bear no meaning, and therefore no information. The meaning (the information) is produced in the mind of a receiver of the signal. Sometimes, we may refer to it as a consciousness, even though this is not exactly true as we will see below. This is because the consciousness itself is a product of mind, and is just one of the concepts.

One has to distinguish data, message, or signal on one side and information on the other. The so-called Shannon information is actually one of the former. While Claude Shannon himself has not defined information, neither in his original work (Shannon1948) nor anywhere else, it is clear that what he occasionally refers to as "information" could be replaced with "data", "message" or "signal", without any harm to the content or significance of his work. The use of word "information "while actually referring to data is highly misleading. It was long understood (MacKay 1969) that information must possess meaning, while data does not. The further history of the discussion can be found in the review by Logan (Logan 2012).

Information is certainly somewhat related to data, signal or message but it is far from being equivalent to them. The relation is that data, message, or signal may (or may not) serve as triggers for information to be produced.

The main difference between information and data is that information is a meaning that a particular mind (consciousness) obtains from data (or encoded message, or signal), while the latter may have no meaning for anyone at all. The data exist on their own, without any "recipient". On the contrary, information can only exist within the mind (of recipient[1]), and moreover it is a product of mind (consciousness).

2. Concepts as elementary constituents of information and consciousness

As explained recently (Rusakov 2022, 2023), concepts are everything human consciousness consists of and operates with. They are elementary constituents of human consciousness. While consciousness in general is a vision, its human part is a vision of "what is not there", i.e. of entities, properties, features, not existing in the real (material, physical) world. We call them concepts. Being abstractions, the concepts, nevertheless, are very real physical entities living in human brain (nervous system in general) in the form of bio-sensory images. Each concept has a unique biochemical portrait (image) that is sensed by the whole body. This duality opens an

[1] This is not exactly correct. Word 'recipient' implies that there is an external signal. However, for information to be produced no external signal is required. It is constantly produced by our nervous system with or without any external signals.

opportunity for empirical study of human consciousness, of information it produces, and one day may even lead to things that were previously unthinkable, such as mind reading and influencing, or implantation of concepts as a way to accelerate learning and extending human intellectual abilities to unimaginable levels.

Conceptualization is the first sign of abstract thinking. Actually, abstract thinking and conceptualization is the same thing. Concepts do not exist in nature. They exist only in the human mind, and are acquired in the process of learning. In addition, even in human consciousness, they are not present at birth and begin to appear much later, after some period of training.

One of the most critical properties of a concept is that it cannot exist without notation, which in our case is language. Any concept must be denoted by a word or phrase. Without notation, the concept does not exist. A concept's name is an integral part of it and allows it to be transferred from one object to another. Certainly, language is not the only means of notation. The notation could be (and apparently was before the emergence of language) an image, a drawing, a meme, a sign, a sound, or similar. Nevertheless, language is the most versatile notation of all, and we are fortunate to have it.

As Harari pointed out (Harari 2015) such concepts as money, religion, nation, state, national borders, banks, and corporations do not correspond to anything at all in nature. They are human inventions. Likewise, not only these but **every** word, every phrase we use refers to a concept created by a human. Even "mother" and "father", even "apple" and "tree", and even the very word "even", are concepts, even though an apple, a tree, or parents are not fictions.

Our speech is made up of concepts. Being an abstraction, each concept has a different meaning for different consciousnesses, depending on the experience and the whole history of development of the particular consciousness. Sometimes the difference is very small and the conversation partners understand each other well, but sometimes the difference is so huge that the listener does not understand the speaker at all, or understands something completely different to what the speaker assumes. This applies not only to such complex concepts as "I think", where the speaker, if properly questioned, would have to admit that (s)he does not understand the meaning of what was said, but also to seemingly very simple, everyday things.

The concepts that make up our consciousness, information and our speech are different not only in different consciousnesses but even in one particular consciousness at different times. One of the most striking and evident examples of such differences is our perception of colors, that changes with luminosity, surrounding colors and many other factors, and that everybody can test on their own consciousness.

3. Definition of information

Thus, information is produced by human consciousness[2] and by nothing else. In the process of learning, human mind creates concepts that serve as perceptual templates for production of information. We may consider these templates as questionnaires, or surveys, that consciousness fills out by scanning its sensors, and constantly updates the templates based on the results. We all know perfectly well that people of different political views produce completely different information from the very same message they receive. This is because their questionnaires have completely different concepts and therefore make completely different requests when analyzing a message.

Information is a result of **analysis** of the conceptual composition of the response. The analysis means decomposition of a signal into conceptual components. Thus, **information is a conceptogram of a signal**, i.e. a set of concepts. Mathematically, information is a vector of concepts.

While not exactly true (see below) one can imagine that consciousness decomposes the signal (or more precisely, the response) into all possible concepts (and their values), just as a medical laboratory analyzes your blood sample and provides you with its results, such as the level of glucose, the level of red blood cells, the level of cholesterol, etc. At the same time, the rest of the body perceives these results through tabulated sensations (each concept has its own sensation). I believe that all this is also accompanied by visual images.

In everyday language, **information is a meaning of analyzed signal for a particular consciousness.**

But at the same time, it is necessary to understand that 1) it is not at all about the "received" signal, and not always about the response to the received signal (which can be completely arbitrary, and even absent), but about the excitation of neural networks that is produced by our brain, both arbitrarily and as a result of "receiving" an external signal; 2) "meaning" is a set of concepts with the values assigned to them in the case of the signal received; 3) Interpreting is the process of decomposing the signal into concepts and assigning values. It is clear that the result of interpretation is different for different consciousness, as well as for the same consciousness at a different time.

One can only imagine how many concepts we have not yet come up with, and therefore how much new information people of the future will find in the same signal? Most likely they will conclude that we knew nothing or were wrong in everything.

Information, as well as each individual concept, is identified by the totality of sensations it evokes. We call it sensory image. Sensory image is a physical fingerprint of a concept in our nervous system. Apparently it corresponds to a certain functional cluster of a neural network.

[2] Unfortunately, due to deceptive nature of our language (Rusakov 2023) we have to use such forms for lack of better alternative. In fact, consciousness (or mind) is a concept itself and thus is a product of our imagination. Therefore it cannot "produce" anything. It is our brain and /or nervous system that do the job.

The concept "I" is one such cluster which only function is to deceive itself and others into believing that "I" is a controller of all the processes happening in the brain and the body. It is so clever in doing it that we don't even question ourselves who is then in charge when "I" is clearly absent (while for example being asleep).

4. Empirical study: Conceptom

For the empirical study we must catalog all existing concepts and perform true mapping of the mind. By this I mean not only mapping the areas of human brain that are responsible for certain functions (Carter 2010) but identifying a unique bio-chemical portrait of each concept, so that it can be unmistakably recognized when activated.

Besides, one must account for all existing concepts, much like connectome researchers account for all existing connections (Seung 2013). I would call such a catalog *conceptom*.

Fortunately for such project, the number of concepts is limited. The maximum number of concepts is defined by current level of development of human society as a whole. The number of concepts present at individual level is clearly lower, and depends on development of an individual. I can even suggest measuring IQ in terms of concepts. It is exactly zero for everyone at birth as well as for animals. It was nearly zero for all individuals yet few million years ago, and probably did not exceed just single digit numbers yet 100,000 years ago.

Since each concept has its lingual notation the number of concepts can be roughly estimated as a number of distinct words and phrases in a language. It would be interesting to track history of growth of number of concepts in human societies. If one assumes exponential growth, then number of concepts at the moment t is given by

$$N(t) = a^t,$$

where a is an arbitrary number. Thus if we know $N(t)$ in two distinct points in time, say today (T) and Δt years ago we can deduct how long ago the first concept has appeared:

$$T = \Delta t \frac{\log N(T)}{\log N(T) - \log N(T-\Delta t)} \quad (1)$$

The most important fact for the empirical study is that number of concepts is finite and relatively small. This makes the task of the cataloging quite attainable, with the help of computers of course.

5. Theoretical study

Let us assume that mapping of the mind is completed, and all currently existing concepts are identified and catalogued, and thus we have complete *conceptom*. This would allow a brain monitoring and literally reading what information is produced since information is a set of concepts.

If N is a number of currently existing concepts, then information I is a vector in N-dimensional space of concepts: $I = (i_1, i_2, \dots i_N)$, where each concept $i_k$is a basis vector in this space.

Since an incoming signal can be represented by vector of concepts $A = (a_1, a_2, \dots a_N)$ and the resulting signal (information) by vector of concepts $B = (b_1, b_2, \dots b_N)$, the converting operator can be thought of as an S-matrix, $S = \{s_{ij}\}$, so that in the short form

$$S \cdot A = B \tag{2}$$

and in the full form

$$\begin{pmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{N1} & \cdots & s_{NN} \end{pmatrix} \begin{pmatrix} a_1 \\ \vdots \\ a_N \end{pmatrix} = \begin{pmatrix} b_1 \\ \vdots \\ b_N \end{pmatrix} \tag{2.1}$$

where $b_j = \sum_{i=1}^{N} s_{ij}\, a_i$.

In a few steps we will realize that matrix$S$ is the operator of consciousness. But first let us briefly step back and look at (2.1) again.

One can immediately see that (2.1) does not reflect the whole picture because it neglects a very important elements, namely *correlators* of concepts. Therefore we need to extend vectors such as A and B to *matrices* with the diagonal elements being original elements of the vectors and the non-diagonal elements being correlators of concepts. Let us denote them by $a_{\mu\nu}$ when $\mu \neq \nu$, while $a_{\mu\mu}$=a_{μ} of (2.1). Thus we arrive to the matrix extension of (2.1):

$$\begin{pmatrix} s_{11} & \cdots & s_{1N} \\ \vdots & \ddots & \vdots \\ s_{N1} & \cdots & s_{NN} \end{pmatrix} \begin{pmatrix} a_{11} & \cdots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{N1} & \cdots & a_{NN} \end{pmatrix} = \begin{pmatrix} b_{11} & \cdots & b_{1N} \\ \vdots & \ddots & \vdots \\ b_{N1} & \cdots & b_{NN} \end{pmatrix} \tag{2.2}$$

Non-diagonal elements, the correlators of concepts, correspond to associations, and thus are attributes of human consciousness. The diagonal elements correspond to animal component of consciousness and are pure reactions. I suspect that for animals, matrices S, A and B are purely diagonal or nearly diagonal, so that their non-diagonal elements are either zero or very close to it.

Obviously, correlators of concepts form clusters. Inside a cluster they are close to 1 (highly correlated concepts) while outside a cluster they are close to zero. For example, the concept of "red" must be highly correlated with such concepts as "color", "blue", and(probably to lesser

degree) "hot", and has a very little correlation with such concepts as "and", "sit", or "sound". The latter of course belong to their own clusters.

Equation (2.2) describes one single act of interaction. Operator$S$ acts on information matrix A and transforms it into new information matrix B. This is exactly the "work" we attribute to our consciousness, and therefore we can truly call operator S an operator of consciousness. Besides, we can view (2.2) as a dynamical process and consider B as a new version of A, only at the next moment of time:

$$A = \Psi(t)$$
$$B = \Psi(t + \Delta t)$$

The equation (2) therefore becomes:

$$S(t) \cdot \Psi(t) = \Psi(t + \Delta t) \qquad (2.3)$$

One can also use (2.3) as a mathematical definition of consciousness:

$$S(t) = \Psi(t + \Delta t)\Psi^{-1}(t), \qquad (2.4)$$

where matrixΨ^{-1} denotes an inverse to Ψ.

Without losing generality one can assume that all matrix elements ofΨ, let us denote them$\psi_{\mu\nu}$, are normalized by their maximum values, so that their values fall in the range 0 to 1. This means that they can be written as:

$$\psi_{\mu\nu}(t) = \sin \varphi_{\mu\nu}(t) \qquad (3)$$

Parameter $\varphi_{\mu\nu}$ in this equation can be thought of as a phase of a concept in case of a diagonal element and as a phase of correlation in the non-diagonal case. One can adopt a convention that $\varphi_{\mu\mu} = 0$ when the concept is not active (not present in a given information) and $\varphi_{\mu\mu} = \frac{\pi}{2}$ when it is the most active. Accordingly, $\varphi_{\mu\nu} = 0$ when there is no correlation between concepts μ and ν, while $\varphi_{\mu\nu} = \frac{\pi}{2}$ when the correlation is the strongest.

Knowing $S(t)$, i.e. all $s_{\mu\nu}(t)$, means a complete description of a particular consciousness. We therefore need some sort of equation that would relate consciousness and information.

Let us recall that

$$\Psi(t + \Delta t) = \Psi(t) + \Delta t \frac{\partial}{\partial t}\Psi(t) + ½(\Delta t)^2 \frac{\partial^2}{\partial t^2}\Psi(t) + \ldots \qquad (4)$$

It is reasonable to assume that action of consciousness S on information matrix $\Psi(t)$ must be such that the resulting $\Psi(t + \Delta t)$ is very close to$\Psi(t)$. In other words, function $\Psi(t)$ must be a

near-equilibrium slow-changing function. Therefore, its first derivative is close to zero, while the second derivative can be neglected at all. As a result, (2.3) takes the Schrodinger-like form:

$$(S(t) - \mathbb{1}) \cdot \Psi(t) = \Delta t \, \frac{\partial}{\partial t} \Psi(t) \qquad (5)$$

Here $\mathbb{1}$ is an identity matrix.

I emphasize that it is only a hypothesis, though quite plausible one, that the consciousness works on information like inertia, i.e. in such a way that brings it to equilibrium state. In other words, its first "goal" is to reconcile any new information with the existing one and calm organism down that everything is good and nothing has changed. If the deviation becomes too large to reconcile then the system becomes unstable (non-equilibrium, off-balance), which gives rise to various processes such as feelings (see next section).

Combining (2.4) with (4), we arrive to another form of (5):

$$\Psi(t) = \exp \frac{1}{\Delta t} \int dt \, (S(t) - \mathbb{1}), \qquad (6)$$

which thus relates matrix of information Ψ with operator of consciousness S and demonstrates that information mainly pertains to a change of S (as all diagonal elements that correspond to animal reactions are subtracted).

6. Why does it feel like something?

If one asks yourself what is common between different feelings, you may be surprised to find out that it is feeling of fainting, losing consciousness, either partially or completely. Even a simple kick or acceleration in a car feels through some dizziness. A fear is a feeling that is closely accompanied by almost certainly approaching fainting. Similarly, "mind altering substances" such as alcohol or drugs that are known to cut some brain connectivity produce feelings varying from extreme happiness to extreme sadness or fear. More subtle feelings can be attributed to various degrees of combinations of these.

This observation suggests that the famous 'why does it feel like something' (Chalmers 1995) should be attributed to loss of connectivity, i.e. *partial loss of consciousness*, in case when reconciliation is not immediately achieved.

In other words, if association coefficient for certain concepts μ and ν at certain moment t deviates too far from its normal, equilibrium, value, the system perceives this as a catastrophic event and cuts this link. Mathematically, this means nullification of corresponding $\varphi_{\mu\nu}$: $\varphi_{\mu\nu}(t) = 0$. In terms of connectome (Seung 2013), one of its links gets disconnected or becomes irresponsive. This means partial loss of consciousness, and is literally feels like free fall, or "rug being pulled from under you", or similar. This very often corresponds to the real loss of spatial balance, disorientation, and feels like dizziness. Accordingly, corresponding alarms go off and

produce variety of feelings (i.e. they activate corresponding concepts). In extreme case when the "kick", either physical (mechanical) that disturbs the spatial balance of organism or purely informational that disturbs its mental balance, is perceived as non-reconcilable then the system may even shut down or restart (fainting). While the origin of feelings was comprehensively addressed before (Rusakov 2023), we believe this new observation brings some additional light to the resolution of the 'hard problem of consciousness' (Chalmers 1995).

7. A possible approach to the phase transition

Let us define the equilibrium state for a particular consciousness. The mean value of matrix of information is the average over long period of time (let us call it ∞):

$$\bar{\Psi} = \int_0^\infty dt\, \Psi(t) \tag{7}$$

Let us also define a parameter α of deviation of $\Psi(t)$ from $\bar{\Psi}$, at particular time t, that we would call a "sanity parameter" and that would serve as an indicator of how far the system deviated from its equilibrium:

$$\alpha^2(t) = \frac{1}{N^2} \sum_{\mu,\nu=1}^{N} (\psi_{\mu\nu}(t) - \overline{\psi_{\mu\nu}})^2 \tag{8}$$

The whole "purpose" of consciousness is to keep this parameter close to zero. This means that there are no drastic changes in situation and information, that everything "makes sense" and no adjustments are needed. However, when $\alpha(t)$ exceeds some critical value α_c it signals that something is not right, the information "does not make sense anymore" and the system becomes prone to a phase transition. The sanity parameter α in terms of phase transition is the order parameter.

My hypothesis is (Rusakov 2023) that this phase transition results in a creation of a new concept. Mathematically this means that we have to add a row and a column in the matrix Ψ, and thus its dimension N increases by 1: $N \to N+1$. If this does not bring a sanity parameter below its critical value then the operation needs to be repeated until it does. It may seem counter-intuitive from the first sight that adding a positive term to (8) would decrease the sum, but let us not forget that it is not a simple addition. The purpose of adding a concept is that the whole matrix Ψ gets rearranged so that its particular elements $\psi_{\mu\nu}(t)$ become closer to their mean values $\overline{\psi_{\mu\nu}}$ and thus certain deviations in the sum (8) become close to zero, and the sum decreases.

As it seems to be confirmed by numerous studies of self-organized criticality of the brain and/or neural networks (Bak 1987, Chialvo2010, Deco 2011, Ivanov 1999, Hensen 1998,Hesse 2014, Mora 2011, Nikolic2007, Plenz 2004/2011/2021, Pruessner 2011) the equilibrium state is in fact a near-critical state. In other words, while our nervous system constantly monitors the information and "prefers" that it does not change (so no action is needed) it nevertheless is always ready for the sudden change and if necessary can reconcile new information by creating new concepts.

It is also claimed (see the cited literature) that this criticality of human brain is unique and is not the case for animals. If this is correct then this criticality resolves the mystery of biological pre-requisite of transition from animal to human (Rusakov 2023). Then it becomes clearer how our animal ancestors acquired first concepts. Though the mystery of why and how this criticality has developed in the first place remains to be answered by biologists.

It would be extremely interesting and important to study this phase transition in more details by using methods of statistical physics. Apparently, condensed-matter physicists have figured it out long ago.

8. Instead of Conclusion

The ultimate goal of this approach is to calculate all $\psi_{\mu\nu}(t)$(or, equivalently,$\varphi_{\mu\nu}(t)$) as well as all $s_{\mu\nu}(t)$, or at least their average values.

It could be helpful to realize that matrices of consciousness $S(t)$, as well as matrices of information $\Psi(t)$, form a group since the result of multiplication of any two of the set is also a matrix from the same set. Therefore, averaging over all possible configurations becomes a familiar integral over a group, and thus the corresponding group-theory techniques can be applied.

As a side note, in (2.4) we indirectly implied that consciousness is an imaginary quantity defined as a 'force' behind transformation of information. Thus, while information is quite tangible observable, the consciousness that is supposedly drives this transformation is an illusion, exactly in accordance with Dennett's idea (Dennett 1992, 2007).

Acknowledgements:I am grateful to Alex Kamenev for drawing my attention to works on self-organized criticality in brain and neural networks, and to Lev Neyman for discussing my hypothesis of loss of connectivity and balance as possible explanation for feelings.

Received April 22, 2024; Accepted May 19, 2024

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