Article

Mathematical Cognition in the TGD Universe

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Abstract

The emergence of mathematical consciousness has meant an enormous evolutionary leap. Something completely exceptional might have occurred when the apple fell down about 358 years ago. Of course, also the emergence of our species was this kind of event? This raises questions. What really happens in this kind of giant leaps of mathematical consciousness or cognitive consciousness in general. What does being or becoming conscious of a mathematical concept mean? Could one see this kind of event as an emergence of a new reflective level of consciousness? One cannot answer these questions unless one can identify the physical correlates of cognition, in particular mathematical cognition. In the sequel these questions are considered in the TGD framework in which the number theoretic and geometric views of physics are dual to each other.

1 Introduction

When one ponders about consciousness, one sooner or later realizes that the emergence of mathematical consciousness has meant an enormous evolutionary leap. Something completely exceptional seems to have occurred when the apple fell down about 540 years ago. Of course, also the emergence of our species was this kind of event?

This raises questions. What really happens in this kind of giant leaps of mathematical consciousness or cognitive consciousness in general. Is our species even in principle able to answer this kind of questions?

What does being or becoming conscious of a mathematical concept mean? Could one see this kind of event as an emergence of a new reflective level of consciousness? But how to describe this kind of hierarchy of levels of consciousness? What kind of phenomenon, bringing in mind phase transition, took place when humankind became conscious of differential and integral calculus, number theory, algebra, logic? Or did already the emergence of our civilization lead to this even?

One can imagine a more modest goal. What could be the physical correlates of these kinds of cognition. The easy solution of the problem would be that deterministic computations are conscious and one can formally regard any time deterministic time evolution as a computer program. This hypothesis does not however explain anything and is untestable.

Even the understanding of how the basic notions and algorithms are realized consciously at the level of cognitive consciousness seems very difficult in the framework of the recent day physics which has hitherto refused to say anything about conscious experience. Is the existing view of physics enough to meet the challenge?

These challenges look formidable but one can try! Maybe one could say at least something about cognition and mathematical cognition?

1. What are the physical correlates of cognition? Cognition is discrete and finite and cognition represents. Could one identify cognitive representations as a discretization of the sensory world. TGD leads to a number theoretic vision about physics dual to the geometric vision and provides a theory of cognition.

   p-Adic topologies seems to be very natural candidates for the topology of cognition. p-Adic number fields fusing together with reals to what is called adele. One can also defined entire hierarchy

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of adeles induced by algebraic extensions of rationals. There is also a second adele-like structure defined by the union of the p-adic number fields. Two p-adic number fields are glued together at interfaces formed by numbers, which have an expansion in powers of an integer divisible by both primes.

2. Concepts, in particular mathematical concepts, are a key element of cognition. What could be the quantum description of concepts and their emergence. Here standard quantum theory suggests an answer. Classically the field of concept is the set of its instances. In quantum theory wave functions in the set could define the instances of the concept.

3. What gives a concious meaning to the concept? Category theoretical thinking suggests that "arrows" as entanglements between concept and other concepts provide the meaning as state function reduction selecting one particular instance of the rule representd by entanglement. In physics this means quantum measurements.

4. What are the quantum physical correlates for Boolean logic? The Fock states define a Boolean algebra and in TGD framework these states span an infinite-D state space. In zero energy ontology (ZEO) [3, 7] of TGD this leads to a natural realization of Boolean algebras and zero energy state defines a quantum version of Boolean map [2].

5. We learn in school mathematics as associations such as "1+2"→ "3" and the recent successes of GPT have demonstrated how powerful tool associations are. I have consider the possible quantum aspects of AI and GPT in [13, 11]) Could associative rules be represented by using quantum entanglement? This could reduce the development of mathematical understanding to the emergence of entangled states representing correlations as rules.

6. What conscious computation could mean? This requires a theory of consciousness and here TGD inspired theory of consciousness provides an approach. Could state function reduction reducing the entanglement defining associative rule give rise to a conscious experience associated with the association? Could one also imagine some kind of quantum hardware of mathematical consciousness perhaps representing the basic arithmetical operations?

Returning to the motivating question about the emergence of new reflective levels of consciousness and cognition, humans are distinguished from the other species by a highly evolved social organizations and in the TGD Universe the emergence of higher levels of consciousness assignable to social structures could be a central element. Single human *viz.* society would be like a single neuron *viz.* the entire brain. Here the hierarchy of magnetic bodies (MBs) is highly suggestive. The emergence of revolutionary ideas certainly requires a highly developed society with a large and complex MB. MBs involve an onion-like hierarchy of extensions of rationals and their dimension measuring their complexity serves as a kind of IQ. Could the emergence of a new level in this hierarchy give rise to these kinds of revolutionary events?

2 Metamathematical and metaphysical considerations

2.1 Some general ideas about mathematical cognition

Doing mathematics need not be a conscious cognitive activity (or more precisely, conscious at our level of hierarchy of conscious entities). Even idiot savants [1, 4], believed to have no idea about the notion of prime, decompose large integers to prime factors. This process could happen at a subconscious level (predicted by the hierarchy of conscious entities existing in TGD Universe) that is consciously but at a lower level of self hierarchy.

Could the basic arithmetics be carved into the basic structure of the physical world extended to provide physical and mathematical correlates of cognition? I have considered the possility that the so called birational maps are morphisms of cognition [12] the basic arithemetic operations could be analogous to particle reactions [17]. Could the basic building bricks of algorithms, identified as quantum computation-like processes, be realized at some lower level as conscious processes decomposing to sequences of SSFRs?

In school we learn mathematical and various empirical rules as associations without much understanding. Could this occur quite generally. Indeed, von Neumann has said that we do not understand mathematial truths, we get accustomed with them. Could these associations be realized in terms of quantum entanglement? This could provide an extremely general mechanism for conscious mathematical cognition.

What could happen in conscious mathematical cognition? The TGD inspired quantum view would be that these experiences are associated with quantum jumps, state function reductions (SFRs), and in the zero energy ontology (ZEO) of TGD, "small" state function reductions (SSFRs) and that one cannot reduce these SSFRs to anything more fundamental. "Big" SFRs (BSFRs) would in turn mean the emergence of something genuinely new, to an eureka experience. In this view one must accept that one cannot give any formula for the meaning associated with the conscious experience accompanying SFR.

Mathematics involves algorithms consisting of basic steps. In arithmetics arithmetic operations define this kind of basic steps. Computationalists might suggest that the mere deterministic algorithm coded by the Bohr orbit-like space-time surface gives rise to a conscious computation. In the TGD framework, a conscious computation however requires a sequence of SSFRs. Could it be physically possible to realize algorithms as sequences of SSFRs and store the steps as tensor product states? This could be realized statistically by using an ensemble of zero energy states with the same state at the passive boundary of CD.

2.2 Concepts, rules, Boolean logic

2.2.1 Quantum view about the notion of concept

Classical concept is the set of its instances. For instance, "house" as a concept is the set of all houses, the field of the concept. In a good approximation a finite set of houses, which share the property of being "house-like". One could say that "house" as a concept is a function in the set of all objects having value 1 in the subset of houses. The instances of "house" correspond to functions which are non-vanishing only for a single instance of "house".

Quantum states could replace the notion of a classical concept. Quantum concept [7] would be a wave function in the set of instances of the concept in the classical sense. There are very many wave functions of this kind and they can be orthogonalized in very many ways and orthogonal functions correspond to independent instances of the concept. This obviously brings a low of new but in quantum physics one cannot avoid this new.

In the set of classical instances one has also a basis of completely localized wave functions. These define instances in the classical sense. Besides this there are bases for which these wave functions are not localized to a single classical instant. For instance, they could be analogs of momentum eigenstates.

One can define the notion of correlation for the quantum instances of the concept using inner product. Two quantum instances for which the wave functions are orthogonal, are uncorrelated. The wave functions of the orthogonal basis are independent instances.

The notion of correlation is much weaker than implication which is a special case of correlation. Should the view of quantum mathematics be formulated in terms of correlation rather than implication?

2.2.2 Quantum Boolean algebra

Classical Boolean algebra consists of subsets of the set. This notion applies also to the set formed by the instances of a classical concept. Set theoretic union and intersection and inclusion as a property of being subset are the basic notions involved. $A \subset B$ means that B implies A.

1. Many-fermion states associated with a second quantized free fermion field, in the TGD framework basically the second quantized spinor field of $H = M^4 \times CP_2$, defined in the set define a basis of Boolean algebra generated by fermionic creation operators. This leads to a connection between Boolean logic realized in terms of fermionic oscillator operators and geometry since spinor structure, whis is realized in terms of the complexified gamma matrices, is induced by the Kähler structure defined for a Kähler manifold. Anticommuting gamma matrices and their hermitian conjugates define the analogs of fermionic oscillator operators. In TGD, the Kähler manifold is the the "world of classical worlds" consisting space-time surfaces of $= M^4 \times CP_2$ satisfying holography forced by general coordinate invariance.

2. The fermionic Boolean algebra defines logical implication in terms of the inclusion relation.

3. The presence of fermion at a given point of time= constant section of the embedding space $H = M^4 \times CP_2$ or 3-D time=constant section 3-surface of 4-D space-time surface "activates" this point. Induced second quantized fermion field at the space-time surface or its sub-manifolds (partonic orbits, string world sheets, string boundaries) defines a realization for the concept "point of this set". In ZEO, pairs of many-fermion states assignable to the boundaries of CD define correlations.

4. What is new is that for each basis of instances of a concept, one has its own classical Boolean logic. One can say that mathematicians work with wave functions localized to say integers but in quantum context also wave function basis consisting of delocalized wave functions are possible.

   Could this mean something new and genuinely interesting? In quantum physics different wave function bases provide a deeper view of the physics of the system. Could this be true also in mathematics? For instance, could one generalize "$\exists$ an instance x with "$\exists$ a wave function $f(x)$" in the set of instances x.

Note that quantum Boolean logic is different from the quantum logic introduced by von Neumann. Quantum logic can be realized in the Hilbert space of wave functions in the space of classical instances for subspaces of Hilbert space and the quantum Boolean logic for second quantized fermions in the space of classical instances.

2.3 How could mathematical rules become conscious?

Mathematical reasoning relies on rules. Mathematical function abstracts the notion of rule. Entanglement provides a universal mechanism for the formation of mathematical functions as associations $(x, f(x))$ as pairs of states labelled by x and $f(x)$. The graph of function can be realized as a superposition of the pairs $(x, f(x))$.

The conscious evaluation of $f(x)$ for some x, which cannot be fixed before measurement, would mean quantum measurement of position x reducing the entanglement. By repeating this process for an ensemble of similar systems, one would obtain a statistical representation for the graph of the function.

Differentiation serves as a second example. Differentiation involves an operator d/dx and function $f(x)$ as an input. It is possible to realize this rule as a learned association assigning df/dx to $f(x)$. Associative rules in turn can be realized by entangling the pairs $(f(x), df/dx(x))$. The conscious calculation of $f(x)$ for a given x would mean quantum measurement reducing the entanglement. This requires an ensemble since the pair $(x, f(x)$ is not predetermined. The meaning of agiven instance of the association is created in SSFR leading to the localization.

2.4 Number theoretical view of physics and cognition

2.4.1 Adelic physics

The above realization of conscious mathematical cognition in terms of associations based on entanglement is extremely general and could be enough. It would be analogous to a computer realization at the level of programs. One can however consider also the possibility that Nature has realized basic mathematical operations also at the level of hardware so to say, as kinds of interaction vertices.

1. The adelic view [5, 6] of cognition suggests that this might be the case. Reals and various p-adic number fields can be combined to form an adele. Adele is very roughly the Cartesian product of reals and p-adic number fields and analogous to a book-like structure with pages labelled by reals and p-adic number fields, one might speak of a Big Book. There is a hierarchy of extensions of rationals, which in turn defines a hierarchy of extensions of p-adic number fields and adeles. This hierarchy of Big Books gives rise to a Big Library.

2. p-Adic number fields and their extensions serve as correlates of cognition. One can define p-adic variants of space-time surfaces and number theoretical universality requires that they satisfy the same equations as the real space-time surfaces. Generalized space-time surfaces have an adelic structure.

   The extensions of p-adic number fields have an algebraic dimension, which can be arbitrarily high, and an attractive idea is that the extensions of p-adic numbers make it possible to imagine higher dimensional spaces.

3. Here one must of course consider the possibility that adelization makes sense only at the level of M^8, which defines the number theoretic view of TGD as algebraic physics involving at the fundamental level only polynomials with integer coefficients smaller than the degree of the polynomial.

Recently I discovered also another hierarchy of book-like structures besides adeles [17].

1. For this generalization of the number concept, for a given extension of rationals the Cartesian product is replaced by a union of extensions of p-adic number fields intersecting along common numbers defining the back of the book-like structure. One can even form the union of these book-like structures assignable to various extensions of rationals.

2. For subsets consisting of unions of subsets in various p-adic number fields this structure is very similar to adeles since the union of this kind of sets has natural representation as a Cartesian product assuming Bose-Einstein or Fermi statistics.

   The 3-D states at either boundary of CD localized to sub-WCW correspond to quantum instances of concepts. Assume that states at both boundaries of CD [15] are eigenstates of the same observables and the states at the passive boundary are fixed. Zero energy states as pairs superpositions of 3-D states at boundaries of CD can be regarded as axiom-like statements A→ B_i, where B_i varies (or their superpositions) allowed by laws of Nature. A milder interpretation is as a correlation of the concepts A and B_i.

3. What is different is that one can speak of boundaries between two p-adic number fields Q_{p_1} and Q_{p_2} consisting of numbers which are expressible as power series of integer divisible by p_1p_2 considered. The algebraic integers of different extensions of the p-adic number fields intersect along integers which have common prime factors. This means multiple p-adicity. Spin glass energy landscape could serve as a physical realization of this structure. This makes it possible to speak of quantum transitions in which a system transforms from asectr p_1 to sector p_2 and the amplitude for the transition corresponds to the overlap in the interface.

2.4.2 $M^8 - H$ duality

Space-time surfaces in $H = M^4 \times CP_2$ indeed realize a differential geometric view of physics whereas the number theoretic view is realized in terms of 4-surfaces in M^8_c.

1. $M^8 - H$ duality [8, 9] relates the number theoretic and differential geometric visions of physics. The notion of derivative is a key notion in differential calculus and of differential geometry. The notion of tangent space having as algebraic counterpart the notion of gradient. The number theoretic holography is purely algebraic concept: everything is algebraic as far as 3-D data for the number theoretic holography is considered.

2. M^8 duality mapping Y^4 to $X^4 \subset H$ is needed to end up with a differential geometric view of physics. This duality generalizes the momentum-position duality for point-like particles, which are now replaced with 3-D surfaces. This duality assigns to a given point of $Y^4 \subset M^8_c$ a point of $M^4 \subset H$ by inversion of the real part its projection to $M^4_c \subset M^8_c$ and the points of CP_2 characterizing its quaternionic normal space containing a commutative 2-D sub-space. These 2-D subspaces form an integrable distribution.

3. CP_2 points parametrize associative/quaternionic normal spaces containing a commutative 2-D subspace and therefore also the tangent spaces of Y^4. Can one say that realizes the notion of tangent in a quaternionic sense? Could classical induced gauge potentials and induced metric represent physically the notion of 4-D normal space of Y^4 determining tangent space as its orthogonal complement. Could this geometric realization of tangent space provide physical and sensory correlates for the experienced notion of tangent?

There are reasons to believe that $M^8 - H$ duality realizes physically the Langlands duality relating number theory and differential geometry is in question.

2.4.3 Could conscious arithmetics be realized at the level of hardware?

Could Nature realize arithmetics at the level of hardware, so to say? Classical Bohr orbit representation is not enough for conscious mathematical operations. The condition that the process is conscious requires a sequence of SSFRs.

Adelization leads to the idea about the physical representation of the basic arithmetic operations as analogs of particle vertices. This generalizes to more general algebraic operations $(A \times B) \to C = A \circ B$.

1. The first notion is cognitive representation. Cognitive representation consists of points of the 4-surface of M^8, or rather 3-surfaces defined by 3-D mass shells H^3, which have M^4 coordinates which are algebraic integers. These selected "active" points carry a fermion (or antifermion) and fermion number 1 at these points is analogous to the characteristic function of the subset telling whether the point belongs to the subset.

2. Arithmetic operations could be seen as analogs of particle reactions. For instance fermion and antifermion characterized by conserved momenta with components as algebraic integers fuse to a boson which the momentum is some of these momenta. This corresponds to addition. Also the time reversal of this process is possible as "co-addition" which gives superposition of possible decompositions to summands.

3. What about multiplication? Supersymmetric arithmetic quantum field theory in which primes label the single particle states would transform realized multiplicative conservation laws to additive since logarithms of primes define additive quantum numbers. Multiplication of numbers could correspond to a consciously experienced sequence of reactions in which particles fuse to a state labelled by a product of primes. The finding of prime factors would be a reversal of this process as a conscious decay process in which a "particle" labelled by an integer decomposes to "particles" labelled by the prime factors.

   In supersymmetric arithmetic field theory the conserved momentum corresponds to a logarithm of an integer. It is not clear whether there is physical representation for these momenta. Note that the phase factors for plane waves provide a multiplicative representation of additive momenta.

Could also quaternionic and even (non-associative) octonionic integer arithmetics be realized in Nature? It would seem that associativity as the basic principle of number theoretic holography restricts the consideration to quaternions.

1. Could supersymmetric arithmetic QFT generalize to quaternions so that multiplication of quaternions would make sense? Quaternionic primes indeed exist and the basic rule would be that only allowed processes involve multiplications realized as many-particle reactions. The interpretation of the quaternions as momenta is however not possible.

2. For arithmetic quantum theory the logarithms of integers define additive momenta. One can represent quaternion q as product of its norm $|q|$ and exponential $exp(\phi)$ of purely imaginary quaternion ϕ defining the analog of a phase of a complex number and in this way one can define the notion of logarithm for quaternion. The analog of the four-momentum spectrum would consist of the sum $log(vertq|) + \phi$ of the logarithm of the norm and imaginary quaternion defining the phase factor. The product of quaternionic phase factors $exp(\phi_i)$ is not not a phase factor defined by the sum of the phases ϕ_i but a product of SO(3) elements. Hence 4-momentum is not conserved although the real part of momentum analogous to energy is conserved. This reflects the breaking of translational symmetry to a rotational symmetry.

2.5 Laws of Nature as counterparts of axioms in ZEO

Mathematical theories rely on a set of axioms from which theorems are deduced using Boolean logic. In the physics inspired picture, the allowed zero energy states as spinor fields in the "world of classical worlds" (WCW) are superpositions of space-time surfaces obeying holography and having coefficients which are pairs of many-fermion states. They would define instances for the Laws of Physics as counterparts of theorems in mathematics.

2.5.1 Classical holography and classical laws of Nature

TGD is analogous to wave mechanics but also differs from it in several ways. Point-like particles are replaced with 3-surfaces and 4-D general coordinate invariance forces their orbits to satisfy holography realized in terms of generalized holomorphy. Analogs of Bohr orbits are in question.

The 3-D configuration space of point-like particles is for given causal diamond (CD) replaced with the union of 4-D Bohr orbits that have ends at the boundaries of CD. This is forced by the small failure of strict determinism for these Bohr orbits.

This has several profound implications. Classical physics becomes an exact part of quantum physics. Path integral disappears and is replaced with a functional integral over the Bohr orbits. The exponent of Kähler function is a non-local functional of the Bohr orbit so that the local divergences of QFTs are absent. TGD is essentially a generalization of wave mechanics. This leads to zero energy ontology (ZEO) [3, 7], which resolves the basic problem of quantum measurement theory.

ZEO is analogous with both computationalism (Bohr orbit is analogous to a deterministic computer program) and biology (Bohr orbit a classical analogy biological function and by holography the structure determines function almost uniquely).

Bohr orbits are instances of classical laws of physics characterized by a variational principle defined by an action defining Kähler function of WCW and strengthened by holography forced by 4-D general coordinate invariance. The holomorphic realization of holography actually implies that Bohr orbits are minimal surfaces for any general coordinate invariant action constructed in terms of the induced geometry [16, 14].

2.5.2 Quantum mechanical holography based on ZEO and quantum laws of Nature

It is not quite straightforward to understand what holography could mean quantum mechanically. For given Bohr orbit, zero energy states can be regarded as many-fermion states at the ends of the space-time surface. They are created by the creation operators of the second quantized spinor fields on H. Modified Dirac equation holds true and the modified Dirac operator is fixed by the action principle defining the space-time surfaces. One can select the fermion states at the ends of the space-time surface freely apart from the constraints posed by the conservation laws for the modified Dirac equation induced by the Dirac equation in H.

One must distinguish between the passive and active boundaries of CD [15]. The passive boundary of CD and 3-D fermion states at it remain unaffected in the sequence of "small" state function reductions (SSFRs), which corresponds to a repeated measurements of same observables whose eigenstates the states at the passive boundary are: this sequence defines self as a conscious entity. In standard wave mechanics one would have Zeno effect. Now the Zeno effect takes place only for the passive boundary of CD since both the active boundary of CD and states at it change in quantum measurements. Active and passive boundaries have the same total conserved quantum numbers. There are also non-conserved quantum numbers such as particle numbers.

Zero energy state defines analog the of S-matrix (I have used also the term U-matrix) as entanglement coefficients between initial state and final state represented in the same basis. The instances for the Laws of Nature are represented as zero energy states A$\rightarrow$ B in the fermionic sector for a given space-time surface.

Superposition of pairs of states and entanglement is possible in principle and would be analogous to a kind of a square root of thermodynamic state. It is however not clear whether the time entanglement is possible. BSFRs could always produce a state which has no time-like entanglement. One would have a superposition of fermionic states with a fixed initial state and varying final fermionic states in the basis used.

2.5.3 The interpretation of ZEO in terms of cognition

In quantum measurement the density matrix is a universal observable. Also the observables commuting with it can be measured in SFR. What these additional observables are? There exist a large number of choices for the decomposition of a system to a subsystem and its complement. Is the choice of the sub-system-complement pair random or guided by some principle?

1. I have considered the possibility that the negentropy gain, defined in terms of the p-adic negentropy, is maximized in SSFR: this makes sense in p-adic context. This however looks rather complex. One can also consider the possibility that if the entanglement coefficients define an extension of rationals which is larger than than associated with the space-time surface itself, entanglement is theoretically stable. It must be admitted that the situation remains open in this respect.

2. An empirical fact is that in the usual quantum measurement there is a clear difference between the measurer and measured. TGD predicts a hierarchy of conscious entities having as counterpart the evolutionary hierarchy of extensions of rational. Could it be that the measurer-measured pairs are always this kind of pairs for which the measured system corresponds to an included extension of rationals? These kinds of quantum measurement hierarchies emerge naturally from the decomposition of Galois extension of rationals to an inclusion hierarchy [10].

   This also suggests a solution to the problem of identifying the observables. The Galois group decomposes to a hierarchy of normal subgroups and its irreps can be factorized to a superposition of entangled products of the irresps for these normal subgroups. Maybe this solves the problem of identifying the universal observables.

It can also happen that the set of the measured observables changes by external perturbations entangling the system with the environment. This could occur for the passive or active boundary of CD. In this kind of situation the state at the passive boundary changes and this would lead to BSFR.

1. In SSFR following a unitary evolution during which CD in statistical sense increases in size, a superposition of A→ B_i is replaced with A→ C_j. S-matrix is assigned with these transitions. One can say that these two "logical" statements correlate. SSFR produces statements about statements. Transition to a higher level in the abstraction hierarchy.

2. The density matrix for the entanglement with the environment is a universal observable and it might change so that it does not commute with the measured internal observables anymore. This could force a "big" SFR (BSFR). In BSFR the arrow of time changes. The roles of boundaries of CD are changed. $\sum_i$ A→ B_i is replaced with time reversed state $\sum_j B_{i_0} \rightarrow C_k$.

3. Could the entanglement with the environment be preserved while self measurements continue? There is no obvious reason preventing this. QCD might provide an example of this situation: hadrons could correspond to higher levels evolving by SSFRs whereas at the quark level SSFRs and BSFRs would occur but commute with the entanglement of quarks which the hadron level. If the observables for the self measurements commute with the density matrix with the environment this seems to be possible. This would fit very nicely with various hierarchies characterizing the TGD Universe.

4. TGD indeed predicts various hierarchies, in particular the hierarchy of space-time sheets and field bodies labelled by the values of h_{eff} having an interpretation in terms of the dimension of extension of rationals associated with the space-time region in question. The field bodies at higher level serve as analogs of the environment for the field bodies at the lower level.

5. The entanglement of a given level with a higher level defines an instance of entanglement with the environment, which is not encountered in standard quantum theory. The extensions of rationals defined by the functional composition of polynomials P defining the 3-D data for number theoreric holography is an example of this kind of hierarchy discussed earlier.

   For two systems at different levels of the hierarchy, the self measurements could be possible if the observables commute with the density matrix associated with this sub-sytem-complement pair. The ordinary quantum measurement theory however suggests that the measurer is always at a higher cognitive level than the measured system: typically the MB of the system would measure the system. Clearly, the number theoretical hierarchies brings in a lot of complexity not encountered in the usual situation and many questions remain to be answered.

2.5.4 Abstraction process as a formation of statements about statements about statements

One can consider both internal measurements, that is self measurements reducing entanglement for a subsystem and its complement and measurements reducing the entanglement between the system and its environment.

The interpretation of SSFRs would be as follows.

1. Measurement for a set of observables commuting with those measured at the passive boundary is performed for the active boundary of CD as for the passive boundary of CD as long as this is possible. Otherwise the state at the passive boundary changes and BSFR occurs. One can call these measurements self measurements.

2. At the active boundary of CD, any decomposition for the system to a subsystem and its complement is possible and the density matrix characterizes this subsystem. The density matrix must commute with the measured observables: this should be be part of the definition of the entanglement coefficients of the zero energy state defining U-matrix, which should be determined more or less uniquely by the maximal symmetries of WCW: recall that zero energy state is amode of WCW spinor field.

   Could the formation of a memory/representation of the correlations defined by the SSFRs as tensor product states formed by $\sum_i$ A→ B_i and $\sum_i$ A→ C_i be a fundamental process of Nature? Could one have an ensemble about a large number of SSFR sequences representing selves. Is an arbitrary many tensor product factors possible? This could give a representation of dynamics as a state.

This would give a hierarchy of statements about statements. At the bottom one would have 3-D states as instances of a concept at the 3-D boundary of CD and zero energy states would represent correlations between concepts.

3. Could the formation of a memory/representation of these correlations as tensor product states formed by SSFR sequences $\sum_i \mathrm{A} \rightarrow \mathrm{B}_i$ with arbitrary number of SSFR be represented as an ensemble be realized?

   It would seem that this can be realized if one has an ensemble in the sense that the passive boundary and state associated with it are identical for these systems. Each of these systems can evolve independently and gives rise to an ensemble of different zero energy states with the same passive state but with different states of evolution depending on the number of SSFRs.

   These zero energy states can interact and entangle and this would give rise to the concept of higher level zero energy states with a fixed passive part. Quantum measurement of the density matrix characterizing this entanglement between two members of the ensemble would give rise to instances of a Law of Nature. This is possible if the self-measurements can continue when the system is entangled with the environment.

   Also the zero many particle zero energy state defined by the ensemble evolves by SSFRs and gives a state at the higher level of hierarchy. One would have a hierarchy of zero energy states as a representation of the abstraction hierarchy.

3 What happened when Newtonian physics emerged?

We are ready to return to the motivating question: Did something very special happen as Newtonian physics emerged? This event was a culmination of a process involving emergence of a heliocentric world view (Copernicus and Galileo). Galilei was the first experimentalist and this meant the emergence of an idea about the existence of fixed objective reality, which became challenged by quantum physics. Galilei experimentally discovered what Einstein called the Equivalence Principle. On the basis of the work of Tyko Brahe, Kepler managed to formulate his laws. Eventually Newton managed to formulate mechanics involving new notions like force, inertial mass, Newton's laws and theory of gravitation.

This required differential calculus. Derivative as a limit and allowing approximation looks completely trivial for physics students but from the point of view of mathematical cognition the situation is different. The notion of limit leads from rationals and algebraic numbers to real numbers and calculus.

If we take seriously the ideas about formation of rules as superposition of state pairs $(x, f(x))$ as the instances of a rule, we must understand how the idea about the derivative of function could emerge from discrete cognitive representations. Is the p-adic view of cognition, based on cognitive representation as a discretization consisting of points in the intersection of algebraic extension of rationals and extensions of p-adic number fields, really enough?

Could one think that the discovery of transcendentals and continuum took place in steps in which transcendental extensions of rationals emerged first. Could it be that the roots of polynomials, which are algebraic numbers, were replaced by those of more general analytic functions so that the roots generated transcendental extension of rationals. Generic transcendentals would have emerged one by one as infinite-dimensional extensions of rationals. This does not conform with the finiteness of cognition. A very different kind of consciousness, transcending the discrete cognitive consciousness, would be in question, was it really so?

In a special role are the extensions by roots of e since they induce finite extensions of p-adic number fields and therefore conform with the finiteness of cognition. Roots of unity lead to discrete Fourier analysis and algebraization of integration.

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