Article

Methods and Applications of Non-Linear Analysis in Neurology and Psycho-physiology

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ABSTRACT

In the light of the results obtained during the last two decades in analysis of signals by time series, it has become evident that the tools of non linear dynamics have their elective role of application in biological, and, in particular, in neuro-physiological and psycho-physiological studies. The basic concept in non linear analysis of experimental time series is that one of recurrence whose conceptual counterpart is represented from variedness and variability that are the foundations of complexity in dynamic processes. Thus, the recurrence plots and the Recurrence Quantification Analysis (RQA) are discussed. It is shown that RQA represents the most general and correct methodology in investigation of experimental time series. By it we arrive to inspect the inner structure of the time series connected to the signals under investigation. Linked to RQA we prospect also the method CZF, recently introduced by us. It is able to account for a true estimation of variability of signals in time as well as in frequency domain. And, consequently, it may be used in conjunction with classical Fourier analysis, accounting however that it is inappropriate in analysis of non linear and non stationary experimental time series. The use of CZF method in fractal analysis is also considered in addition to standard index as Hurst exponent. A large field of possible applications in neurological as well as in psycho-physiological studies is given. Also, there are given examples of other and (possibly linked) applications as example the analysis of beat-to-beat fluctuations of human heartbeat intervals that is sovereign in psycho-physiological studies. We give applications on some different planes to evidence the particular sensitivity of such methods. We reach the objective to show that the previously exposed methods are also able to predict in advance the advent of ventricular tachycardia and/or of ventricular fibrillation. The RQA analysis gives good results. The CZF method gives the most excellent results showing that it is able to give very significant indexes of prediction. We also apply such methods in investigation of state anxiety, and proposing in detail a quantum like model of such phenomenological status of the mind.

Key Words: non-linear analysis, time series, neurology, psycho-physiology, RQA, CZF, anxiety, quantum-like, mind.

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1. The Recurrences and the Variability of Signals in Nature

Only few systems in Nature exhibit linearity. The greatest whole of natural systems, especially those who pertain to biological matter, to physiological (neuro-physiological and psycho-physiological) and to psychological processes, possess a complexity that results in a great variedness and variability, linked to non linearity, to non stationarity, and to non predictability of their time dynamics. In the time domain, traditional methods were first used to describe the amplitude distribution of signals and later, methodologies used spectral analysis methods. However, they suffer of fundamental limits. They are applied assuming linearity and stationarity of signals that actually do not exist. The consequence is that such methods are unable to analyse in a proper way the irregularity present in most of signals. The results show that such irregularity is at the basis of the dynamics that we intend to explore. It reveals that complex behaviours of the system are very distant from previously accepted principles as it is the case, as example for biological signals, on the view of homeostatic equilibrium and of other similar mechanism of controls. The study of this very irregular behaviour requires the introduction of new basic principles. Therefore, nonlinear science is becoming an emerging methodological and theoretical framework that makes up what is called the science of the complexity, often called also chaos theory.

2. The Chaos Theory

The aim of non linear methodologies is a description of complexity and the exploration of the multidimensional interactions within and among components of given systems. An important concept here is that of chaotic behaviour. It will be defined chaotic if trajectories issuing from points of whatever degree of proximity in the space of phase, distance themselves from one another over time in an exponential way.

In detail, the basic critical principles may be reassumed as it follows:

1) Non linear systems under certain conditions may exhibit chaotic behaviour;
2) The behaviour of a chaotic system can change drastically in response to small changes in the system's initial conditions;
3) A chaotic system is deterministic;
4) In chaotic systems the output system is no more proportionate to system input.

Chaos may be identified in systems also excluding the requirement of determinism. The standard approach to classical dynamics assumes the Laplace point of view that the time evolution of a system is uniquely determined by it's initial conditions. Existence and uniqueness theorem of differential equations require that the equations of motion everywhere satisfy the Lipschitz condition. It has long been tacitly assumed that Nature is deterministic, and that correspondingly, the equations of motion describing physical systems are Lipschitz. However, there is no a priori reason to believe that Nature is unfailingly Lipschitzian. In very different conditions of interest, some systems exhibit physical solutions corresponding to equations of motion that violate the Lipschitz condition. The point is of particular interest. If a dynamical system is non-Lipschitz at a singular point, it is possible that several solutions will intersect at this point. This singularity is a common point among many trajectories, and the dynamics of the system, after the singular point is intersected, is not in any way determined by the dynamics before. Hence the term non-deterministic dynamics takes place. For a non-deterministic system, it is entirely possible (if not likely) that as the various solutions move away from the singularity, they will evolve very differently, and tend to diverge. Several solutions coincide at the non-Lipschitz singularity, and therefore whenever a phase space trajectory comes near this point, any arbitrarily small perturbation may push the trajectory on to a completely different solution. As noise is intrinsic to any physical system, the time evolution of a non-deterministic dynamical system will consist of a series of transient trajectories, with a new one being chosen randomly whenever the solution (in the presence of noise) nears the non-Lipschitz point. We term such behaviour non-deterministic chaos. This approach to chaos theory was initiated by Zak, Zbilut and Webber [1] and rather recently we have given several examples, theoretical and experimental verifications on this important chaotic behaviour [2].

2.1 Embedding time series in phase space

The notion of phase space is well known in physics. Let us consider a system, determined by the set of its variables. Since they are known, those values specify the state of the system at any time. We may represent one set of those values as a point in a space, with coordinates corresponding to those variables. This construction of space is called phase space. The set of states of the system is represented by the set of points in the phase space. The question of interest is that we perform an analysis of the topological properties of phase space but, as a counterpart, we obtain insights into the dynamic nature of the system. In experimental conditions, especially in experimental clinical studies, we are unable to measure all the variables of the system. In this case we may be able to reconstruct equally a phase space from experimental data where only one of the present variables (characterizing the whole system) is actually measured. The phase space is realized by a set of independent coordinates. Generally speaking, the attractor is the phase space set generated by a dynamical system represented by a set of difference or differential equations. In the actual case, let us take a non linear dynamical system represented by three independent variables $X(t), Y(t), Z(t)$, functions of time. The phase space set is given by the values of the variables at each time. The point (x, y, z) in phase space gives the values of the three variables and thus the state of the system at each time. Usually, in physics, for example, we plot one of the variables and its derivatives,

$$X,\ \frac{dX}{dt},\ \frac{d^2X}{dt^2},\ \ldots. \tag{2.1}$$

on the three perpendicular axes (x, y, z). The result is that we have reconstructed the phase space using only one of the three time series using also the derivatives of $X(t)$. This is a licit step since $Y(t)$ and $Z(t)$ are coupled to $X(t)$ through non linear equations. Consider that in experiments we have a fixed time sampling, Δt (time series recorded at equal time intervals), and the time series is given in the following manner

$$X(0), X(\Delta t), X(2\Delta t), X(3\Delta t), \ldots\ldots\ldots\ldots, X(n\Delta t) \tag{2.2}$$

We could also differentiate such values determining $dX/dt, d^2X/dt^2$, …..but such a procedure is unprofitable. In fact, also if our time series data should contain only very small errors in measurements, they should become larger errors during such operations. We may follow another procedure. We introduce a time lag $\tau = m\Delta t$ and consider each point in phase space, given by the following vector expression

$$\left[X(t), X(t+\tau), X(t+2\tau), X(t+3\tau), \ldots\ldots X(t+(N-1)\tau)\right] = \overline{X}_N(t) \qquad (2.3)$$

where N is the selected dimension of the phase space. Note that assuming such a procedure in phase space reconstruction we do not lose generality since, as it is easy to show, the coordinates of the phase space reconstructed in this manner, using time delays, are linear combinations of the derivatives.

This procedure of reconstruction of phase space starting with the given time series is called *embedding*. This is the method presently used for reconstruction of phase space of experimentally sampled time series. Takens in 1981 [4] showed that this embedding method, based on time lags, is certainly valid under some suitable conditions. The first requirement is that the considered time series must be twice differentiable. If this requirement is not satisfied, and it happens often, when the considered time series is a fractal, the fractal dimension, calculated by the embedding method, may also not be equal to the true fractal dimension of the phase space set. Still, the other statement relating Takens theorem, requires that in a realistic reconstruction of phase space, say of dimension D, we must embed in a space of dimension $(2D+1)$ in order to express enough dimensions. This is to avoid the possibility that the $N-$dimensional orbits intersect themselves in a false manner.

2.2 The Determination of Time Lag τ

Some different procedures may be followed to determine the time lag of the given time series in the embedding method. There are cases in which the appropriate choice of the time lag is rather simple. In fact, it may be seen from the basic features of the system under consideration. It is rather simple to evaluate the proper time lag if we are investigating physiological processes exhibiting with evidence their natural time scale. In other cases the estimation of the time lag may be not be so simple since we do not have a direct indication of the appropriate time lag. Let us consider, for example, the case of investigation of a physiological process involving electroencephalographic studies.

Experience in methodological analysis of time series often helps to solve such problems. The problem must be solved with particular care. The proper choice of the time lag is of fundamental importance because in chaotic signals the relation between the dimension of an embedding space and real phase space is strongly linked to the length chosen for a time lag. A too large selected time lag will determine unwished noise in embedding and so the observation of the chaotic attractor will be strongly compromised. The use of a too small lag may result in strong correlations among the components of the signal (2.3), and the local geometry of embedding results much like as a line (i.e. dimension equal to 1), and damaging image reconstruction of the chaotic attractor.

As a methodological praxis, it is useful to study the autocorrelation function of the given time series. Given the time series $X(n)$, n=1, 2, ...N, the autocorrelation function, $Au(\tau)$, at lag τ is defined as:

$$Au(\tau) = \frac{1}{N-\tau}\sum_{n=1}^{N-\tau} X(n)X(n+\tau) \qquad (2.4)$$

Values of time series correlate with themselves and the correlation diminishes as the time lag between two points increases. Correlation decreases with time. The time lag is selected as the autocorrelation function reaches its first zero. Often another useful criteria is to take the time lag as the autocorrelation function decreases to $1/e = 0.37$.

In addition to use of the autocorrelation function, one can employ the mutual information content, $MI(\tau)$. Mean mutual information is given in the following manner [5]

$$MI(\tau) = \sum_{X(i),X(i+\tau)} P(X(i), X(i+\tau))\log_2 \frac{P(X(i), X(i+\tau))}{P(X(i))P(X(i+\tau))} \qquad (2.5)$$

The time at which the first local minimum of mutual information content is reached, may represent a good choice for time lag. Both $Au(\tau)$ and $MI(\tau)$ must be used, selecting the time lag provided by $MI(\tau)$ if $Au(\tau)$ and $MI(\tau)$ predict different results. This is preferable since $MI(\tau)$ also accounts for non linear contributions in a time series.

2.3 Embedding Theorem and False Nearest Neighbors

As previously outlined, according to the embedding theorem (see Takens theorem for details), the choice of dimension N of reconstructed phase space should require a priori knowledge of the dimension d_F of the original attractor with $N > 2d_F$. This is decisively unrealistic for time series of experimental data. Selecting N in absence of a given criterion, it may result in too small a choice as compared to the d_F of the original attractor. It is possible to employ what is called the criterion of false nearest neighbors (FNN) in reconstructed phase space [6]. A point of data sets is said to be a FNN when it comprises the local nearest neighbors not actually but only because the orbit is constructed in a too small an embedded space determining its self-crossing. This difficulty may be overcome by adding sufficient coordinates to the embedding space. The criterion to use is to increase N in a step manner until the number of the FNN goes substantially to zero. Usually, a threshold of about 5% may be acceptable. Le us calculate the distance between two points in a selected embedding dimension of N, obtaining the value $D_N(i)$. In the $(N+1)$ embedding dimension, we will have $D_{N+1}(i)$. Such values satisfy the following relation

$$\sqrt{\frac{D_{N+1}^2(i) - D_N^2(i)}{D_N^2(i)}} = \frac{\left|X_{i+N\tau} - X_{i+N\tau}^{NN}\right|^2}{D_N(i)} \qquad (2.6)$$

where NN indicates that we consider a point selected conventionally near a given point. A fixed threshold value is used and step by step it is verified if the (2.6) exceeds or not the prefixed threshold value.

3. Fractality and Non Linearity of Experimental Time Series

3.1 Fractality and Deterministic chaos of Time Series

The use of non linear methods presumes that the signal under study is represented by an experimental time series relating a non linear system. Sometimes it possesses some deterministic features that may be also chaotic and must be investigated by the methodology discussed in the previous sections.

Fractality refers to the features of a given stochastic time series. It shows temporal self-similarity. A time series is said self-similar if its amplitude distribution remains unchanged by a constant factor even when the sampling rate is changed. In the time domain one observes similar patterns at different time scales. In the frequency domain the basic feature of a fractal time series is its power law spectrum in the proper logarithmic scale. Fractals and chaos have many common points. When the phase space set is fractal, the system that generated the time series is chaotic. Chaotic systems can be arranged that generate a phase space set of a given fractal form. However, the systems and the processes studied by fractals and chaos are essentially different. Fractals must be considered processes in which a small section resembles the whole. The point in fractal analysis is to determine if the given experimental time series contains self-similar features. Deterministic chaos means that the output of a non linear deterministic system is so complex that in some manner mimes random behaviour. The point in deterministic chaos analysis is to investigate the given experimental time series that arises from a deterministic process and to understand in some manner the mathematical features of such a process. Regarding a chaotic time series, this means that the corresponding system has sensitivity to initial conditions. When we speak about strange attractors this means that the attractor is fractal [for details see 4]. It is very important to account for such properties since there are also chaotic systems that are not strange in the sense that they are exponentially sensitive to initial conditions but do not have a fractal attractor. Still we have non chaotic systems that are strange in the sense that they are not sensitive to initial conditions but they have a fractal attractor. In conclusion, we must be careful in considering fractals and non linear approaches since they are very different from each other. Often, instead, we are induced to erroneously mix different things with serious mistakes.

The geometry of the attractors is frequently examined by calculating the so called correlation dimension [7]. The self-similar property of the attractor is estimated by the scaling behaviour of the correlation integral

$$C_N(r) = \frac{1}{n^2}\sum_{i \neq j} \vartheta(r - \left\| \overline{X}_N(i) - \overline{X}_N(j) \right\|)$$

where $\vartheta(\cdot)$ is 1 for positive arguments and 0 for negative arguments. For a fixed a sphere of radius r, in the reconstructed phase space $C_N(r)$ gives the normalized number of points in it. For stochastic signals the correlation integral, calculated in the $N-$dimensional space, scales as

$$C_N(r) \approx r^N$$

For bounded signals there is a finite scaling exponent so that

$$C_N(r) \approx r^d \text{ with } d < N.$$

The correlation dimension, usually indicated by D_2, is calculated as the slope of the linear behaviour of $\log r$ vs. $\log C_N(r)$. The value 1.0 is obtained in the case of a limit cycle, 2.0 instead is calculated in the case of a torus. A calculated non- integer value instead indicates that the phase space has a fractal geometry. However, in analysis of experimental time series the calculation of the correlation dimension does not offer results sensitive enough to conclude that for a non-integer, a fractal dimension that could be generated by a deterministic chaotic system. Stochastic signals may mimic chaotic data and furthermore, time series of stationary data are always required. This last requirement is rarely obtained by experimental time series, especially those of biological or physiological interest.

3.2 Estimation of Lyapunov Exponents

As previously mentioned, chaotic systems show a dynamics where phase space trajectories with nearly identical initial states will, however, separate from each other at an exponentially increasing rate. This is usually called the sensitive dependence on initial conditions in chaotic deterministic systems. The spectrum of the Lyapunov exponents captures this basic feature of the dynamics of these systems. We may consider the two nearest neighboring points in phase space at time 0 and at time t. Let us consider also a direction i-th in space. Let $\|\delta x_i(0)\|$ be the distance at time 0 and $\|\delta x_i(t)\|$ the distance at time t. The Lyapunov exponent, λ_i (direction i-th), will be calculated such that [8]

$$\frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|} = e^{\lambda_i t} \qquad \text{for } t \to \infty$$

that is equivalent to

$$\lambda_i = \lim_{t\to\infty} \frac{1}{t} Ln \frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|}$$

It is possible to reconstruct the Lyapunov spectrum accounting for all the directions in phase space. Chaotic systems are characterized by having at least one positive Lyapunov exponent while their sum generally must be negative. Given there is a whole spectrum of Lyapunov exponents, the number of them is equal to the number of dimensions of the phase space. If the system is conservative (i.e. there is no dissipation), a volume element of the phase space will stay the same along a trajectory. Thus the sum of all Lyapunov exponents must be zero. If the system is dissipative, the sum of Lyapunov exponents is negative.

The Lyapunov spectrum can be used also to give an estimate of the rate of entropy production and of the fractal dimension of the considered dynamical system. In particular from the knowledge of the Lyapunov spectrum it is possible to obtain the so-called Kaplan-Yorke dimension D_{KY}, that is defined as follows:

$$D_{KY} = k + \sum_{i=1}^{k} \frac{\lambda_i}{|\lambda_{k+1}|}$$

where k is the maximum integer such that the sum of the k largest exponents is still non-negative. D_{KY} represents an upper bound for the information dimension of the system. Moreover, the sum of all the positive Lyapunov exponents gives an estimate of the Kolmogorov-Sinai entropy accordingly to Pesin's theorem [9]

In conclusion, the Lyapunov exponent is a measure of the rate at which nearby trajectories in phase space diverge. Chaotic orbits show at least one positive Lyapunov exponent. Instead periodic orbits all give negative Lyapunov exponents. It is of interest also the analysis of a Lyapunov exponent equal to zero. It says that we are near a bifurcation.

There is still another feature to outline. It is common to avoid to calculating the whole Lyapunov spectrum, estimating instead only the most positive one, usually refered to as the largest one. A positive value is normally taken as indication that the system is chaotic. The inverse of the largest Lyapunov exponent is sometimes referred to in the literature as Lyapunov time, and defines the characteristic folding time. For chaotic orbits it is finite, whereas for regular orbits it will be infinite. Finally, to quantify predictability of the system, the rate of divergence of the trajectories in phase space must be evaluated by Lyapunov exponents and Kolmogorov-Sinai entropy.

Under the perspective of the analysis one must account for the calculation of Lyapunov exponents from limited experimental data of time series. Various methods have been proposed [10]. Generally speaking, however, these methods may be sensitive to variations in parameters, e.g., number of data points, embedding dimension, reconstructed time delay, and are usually reliable with care.

3.3 The Method of Surrogate Data in Time Series

At this stage of the present exposition, the reader will have realized that the most unfavourable snare in the investigation of experimental time series, possibly chaotic, is that the methods we have at our availability, are inclined to give similar results in the case of deterministic chaotic dynamics and stochastic noise so that distinguishing deterministic chaos from noise becomes an important problem. Starting with a given experimental time series, stochastic surrogate data may be generated having the same power spectra as the original one, but having random phase relationship among the Fourier components. If any numerical procedure in studying deterministic-chaotic dynamics will produce the same results for surrogate data as well as for the original ones within a prefixed criterion, we will not reject the null hypothesis that the analyzed dynamics is determiend by a linear stochastic model rather than to be represneted by deterministic chaos. Often the method of the shuffled data is used. Data of the original time series are shuffled, and this operation preserves the probability distribution but produces generally a very different power spectrum and correlation function.

3.4 Fractional Brownian Analysis in Time Seires

It is well knwn that the study of stochastic processes with power-law spectra started with the celebrated paper on fractional Brownian motion (fBm) by Mandelbrot and Van Ness in 1968 [11]. Fixing the initial conditions, fBm is defined by the following equation

$$X(ht) \overset{d}{=} h^{H} X(t) \tag{3.1}$$

Given a self-similar fractal time series, (3.1) establishes that the distribution remains unchanged by the factor h^H even after the time scale is changed. $(\overset{d}{=})$ states that the statistical distribution function remains unchanged. H is called Hurst exponent, varying as $0 < H < 1$, and it characterizes the general power – law scaling. For an additive process of Gaussian white noise, we have $H = 0.5$. H values greater than 0.5 indicate persistence in time series. This is to say that a past trend persists into the future(long-range correlation). Instead, H values less than 0.5 indicate antipersistence and this is to say that past trends tend to reverse in the future. The fBm also exhibits power-law behaviour in the Fourier spectrum. There is a linear relationship between the log of spectral power vs. log of frequency. The inverse of the slope in the log-log plot is called the spectral exponent β ($1/f^{\beta}$ behaviour), and it is related to H by the following relationship

$$H = \frac{\beta - 1}{2}$$

4. Recurrence Quantification Analysis and the CZF Method

4.1 Introduction

Let us take up some of the concepts exposed in the previous sections. It was outlined that the most important concept in studies of nonlinear processes by time series is that one of recurrence. A recurrence plot is the visualization of a square recurrence matrix of distance elements within a cutoff limit. We outlined also the importance of Takens theorem relating higher dimensional reconstruction of signals by the method of time delay. It is important to reaffirm here that the topological features of a higher dimensional system consisting of multiple coupled variables may be reconstructed from a single measured variable. We measure only one of these variables, and correspondingly we obtain important information on the whole system underlying the dynamics. The reconstruction happens in the phase space. Let us discuss an example previously introduced in [3] to illustrate the importance of the approach.

Let us take a single lead of the ECG recorded signal. We have in this manner an ECG signal in its one dimensional representation of voltage as a function of time. A digitised time series is obtained. Actually ECG derives from summed cardiac potentials that act simultaneously under the frontal, the saggital and the horizontal orthogonal planes, and thus along three dimensions. In order to have an accurate representation of the ECG signal, we need to simultaneously record voltages in time in these three orthogonal planes. However, if we perform a reconstruction plotting 1-dimensional data again itself and twice delayed, that is to say, delayed by τ and 2τ, on a three axis plot, the signal is represented as the reconstructed 3-dimensional space. Topologically, these loops are the same thing as the simultaneous plotting of three orthogonal recorded ECG leads. In the previous sections, we outlined that in order to realize such a methodology we need to estimate properly the time delay and the embedding dimension.

In analysis, recurrence is the most important concept. Of course, variedness and variability relate the complexity of a given dynamics. In recurrence analysis one must define some parameters that are the range, the norm, the rescaling and, finally, the radius, and the line. The range defines a window on the dynamics under investigation, selecting the starting point and the ending point in the time series to be analysed. For the norm, one has to distinguish the minimum norm, the maximum and the Euclidean norms. The norm function geometrically defines the size and the shape of the neighborhood surrounding each reference point. The Euclidean norm defines the Euclidean distance between paired points in phase space. The rescaling relates the fact that the distance matrix can be rescaled by dividing each element in the distance matrix by either the mean distance or maximum distance of the whole matrix. Finally, the radius is expressed in units relative to the elements in the distance matrix, whether or not these elements have been rescaled. The line parameter is decisive when we have to extract quantitative features from recurrence plots We have a length of a recurrence feature and a prefixed line parameter so that such features may be rejected in quantitative analysis if it results are shorter than selected line parameter.

4.2 The Recurrence Quantification Analysis

Recurrence analysis was first introduced by Eckmann, Kamphorst and Ruelle in 1987 [12], A recurrence quantification analysis, indicated by RQA, was subsequently introduced by Zbilut and Webber [13] and further enriched by the introduction of other variables by Marwan [14]. An exceptional element of value of RQA is that this method has no restrictions in its applications: as we will explain later, for example it may be applied also to non stationary time series.

The first recurrence variable is the % Recurrence (%REC). %REC quantifies the percentage of recurrent points falling within the specified radius. Out of any doubt we may define it the most important variable in analysis of time series. The second recurrence variable is the % Determinism (%DET). %DET measures the proportion of recurrent points forming diagonal line structures. Diagonal line segments must have a minimum length in relation to above line parameter. Repeating or deterministic patterns are characterized by this variable. Periodic signals will give long diagonal lines. Instead chaotic signals will give very short diagonal lines. Stochastic signals will not determine diagonal lines unless a very high value of the radius will be selected.

The third recurrence variable is the MaxLine (LMAX). It is the length of the longest diagonal line segment in the plot excluding obviously the main diagonal line of identity. This is a variable of interest since it inversely scales with the most positive Lyapunov exponent previously discussed. Therefore, the shorter the maxline results, the more chaotic the signal is. In addition, RQA may be performed by epochs, so that LMAX enables evaluation of Lyapunov exponent locally.

The other important recurrence variable is entropy (ENT). It relates Shannon information entropy of all the diagonal line lengths distributed over integer bins in a histogram. ENT may be considered a measure of the signal complexity and is given in bits/bin. For simple periodic systems with all diagonal lines of equal length and the entropy is expected to go to zero.

Another decisive variable in RQA is the trend (TND). All the above methods discussed in the previous sections hold for stationary time series. This is a condition rarely met in analysis of experimental time series and especially in the field of biological signals. RQA may be applied for any kind of experimental time series including non stationary time series. This is one of the reasons to appreciate the RQA method. The trend (TND) still quantifies the degree of non stationarity of the time series under investigation. If recurrent points are homogeneously distributed across the recurrence plot, TND values will approach zero. If they are heterogeneously distributed across the recurrence plot, TND values will result different from zero.

The sixth important variable in RQA, introduced by Marwan [14] is %Laminarity (%LAM). %LAM measures the percentage of recurrent points in vertical line structures rather than diagonal line structures. Finally, the Trapping Time (TT) measures the average length of vertical line structures. Square areas (really a combination of vertical and diagonal lines) indicate laminar (singular) areas, possibly intermittency, suggesting transitional regimes, chaos-ordered, chaos-chaos transitions.

In conclusion, RQA may be considered at the moment the most powerful method for analysis of any kind of time series without limitations of any kind. The confirmation is in the large and growing interest in literature for such a methodology over the last decade. Several fields have been explored by RQA from general chaos science to proper fields of application as clinical electro-physiology [see as example 15], molecular dynamics, psychology and mind pathologies [see for example 16], finance, just to list only some of the several fields impacted by this non linear methodology of analysis.

4.3 Further Advances in Analysis of Variability in Time Series: the CZF method

As previously indicated, complexity of natural processes relates the variedness and the variability of the experimentally measured signals in the form of time series. The CZF method relates this feature, and it derives from the surname (Conte, Zbilut, Federici) of the authors who introduced it.

Let us recall an old notion. The presence of an harmonic component in a given time series is revealed by its power spectrum $P(\nu)$ given by the squared norm of the Fourier transform of the given time series $X(t)$ as

$$P(\nu) = \left\| \int_0^\infty e^{i\nu t} X(t) dt \right\|^2 \tag{4.1}$$

and evidences sharp peaks.

FFT (Fast Fourier Transform), in its discrete version, is currently applied in analysis of non linear time series. All we know that, because of its simplicity, Fourier analysis has dominated and still dominates the data analysis efforts. This happens ignoring the fact that FFT is valid under extremely general conditions but essentially under the respect of some crucial restrictions that often result largely violated, especially in the field of the electrophysiological signals. Three stringent conditions must be observed:

1) the system under investigation must be linear.
2) The data of the time series under investigation must be strictly periodic and stationary.
3) All the data of the time series under investigation must be sampled at equally spaced time intervals.

The consequences of such improper use of the FFT are significant. In particular, the presence of non linearity and of non stationarity give little sense to the results that are obtained. Consequently we will discuss now a non linear method, the CZF. It was previously introduced by us in literature [17], and it presents, conceptual links with RQA.

Let us start with Hurst analysis [18] that brings light on some statistical properties of time series $X(t)$ that scale with an observed period of observation T and a time resolution μ. As previously shown, scaling results characterized by an exponent H that relates the long-term statistical dependence of the signal. In substance, one may generalize such Hurst approach, expressing the scaling behaviour of statistically significant properties of the signal. Indicating by E the mean values, we have to analyze the q-order moments of the distribution of the increments

$$K_q(\tau) = \frac{E(\left|X(t+\tau) - X(t)\right|^q}{E(\left|X(t)\right|^q)} \tag{4.2}$$

The (4.2) represents the statistical time evolution of the given stochastic variable $X(t)$.
For q=2, we may re-write the (4.2) in the following manner

$$\gamma(h) = \frac{1}{2n(h)} \sum_{i=1}^{n(h)} \left[X(u_{i+h}) - X(u_i)\right]^2 \tag{4.3}$$

that estimates the variogram of the given time series. Here, $n(h)$ is the number of pairs at lag distance h while $X(u_i)$ and $X(u_{i+h})$ are time sampled series values at times t and $t+h$,

$t = u_1, u_2, ...;\ h = 1,2,3,.....$. In substance, the variogram is a statistical measure expressed in the form:

$$\gamma(h) = \frac{1}{2} Var\left[X(u+h) - X(u)\right] \tag{4.4}$$

The variogram here introduced represents the a valuable measure of complexity in a given non linear time series and at the same time its elaboration enables us to overcome the difficulties previously mentioned for use of the FFT in non stationary and non linear time series. The concept of variability is sovereign in this case. Let us take an example to illustrate its relevance.
Let us admit we have a time series given only by six terms:

$$X_1, X_2, X_3, X_4, X_5, X_6 . \tag{4.5}$$

The first time we select time lag $h = 1$, and using the (4.3) we calculate variability of this signal at this time scale, obtaining:

$$(X_1 - X_2)^2 + (X_2 - X_3)^2 + (X_3 - X_4)^2 + (X_4 - X_5)^2 + (X_5 - X_6)^2 \tag{4.6}$$

This is the variability of the signal at time scale $h = 1$ and, in accord with the (4.3), we indicate it by $\gamma_1(h) \equiv \gamma_1(1)$.

Note some important features:
The differences $(X_i - X_{i+1})^2$ in the (4.6) will account directly for the fluctuations (and thus of the total variability) that intervene in X_{i+1} with respect to X_i. It will be due to the particular features of the dynamics under investigation. Let us consider for example the case of (4.5) representing the beat-to-beat fluctuations of human heartbeat intervals. The (4.6) will represent total variability in time lag $h = 1$ due to the regulative activity exercised by sympathetic, vagal, and VLF activities in the time lag considered. Still, the count of such variability will happen for all the points of the given time series and thus it will account for the total variability at the fixed time scale of resolution for the whole considered R–R process.

Finally, if $\gamma_1(1)$ will assume a value going to zero, we will conclude that at such time scale (time lag delay h = 1) the variability of the signal in this time lag is very modest. Otherwise, if $\gamma_1(1)$ is different from zero in a consistent way, we will conclude that it gives great variability, attributed to the presence of a relevant activity of control. In the same way we will proceed considering for example (4.5) to represent an EEG signal recorded at some electrode at a given sampling frequency. In this case, (4.6) represents the total variability in cerebral activity at the selected electrode and at the time resolution of $h = 1$. After to having computed the total variability of signals at this time resolution $h = 1$, we will continue our calculation evaluating this time the total variability of the signal at the time resolution $h = 2$, and thus calculating $\gamma_2(2)$. In a similar way we will proceed calculating total variability at the time scale resolution corresponding to $h = 3$ and so on, completing the analysis of variability at each time scale. In conclusion we will calculate the final variability of the given signal step by step at different time scales. The result will be a diagram in a plot in which in axis of the ordinate we will have the values of variability (in its corresponding unity of measurement) while in the axis of the abscissa we will have the corresponding value of h, that is to say of the corresponding time resolution.

Note that, in order to calculate the final value of the total variability we may decide (at time lag $h=1$ but so also at the following steps) to divide (4.6) by the number of pairs employed in the calculation. In this manner we will obtain the mean value of variability at such time scale.

To complete our exposition on the CZF method we must still outline that, in calculating $\gamma_i(h)$ we may also use the embedding procedure for reconstruction in phase space and thus performing in this case a more elaborate and significant exploration of the time series under investigation.

From a methodological view point we may still outline that by the CZF method we may perform also fractal analysis of the given time series. In fact we may use the Fractal Variance Function, $\gamma(h)$, and the Generalized Fractal Dimension, $D_{\dim}$, by the following equation

$$\gamma(h) = Ch^{D_{\dim}} \tag{4.7}$$

and finally estimating the Marginal Density Function for self-affine distributions, given by the following equation [19]:

$$P(h) = ak^{-a}h^{a-1} \tag{4.8}$$

This last consideration completes our exposition on CZF method. It remains to be explained the manner in which the CZF method overcomes the difficulties previously noted in the case of FFT and thus the manner in which it must be applied to perform an analysis of variability in the frequency domain. To illustrate such a methodology we will use two basic examples: the first is the case of HRV, that is the analysis of heart rate variability by using time series of R-R intervals from the ECG. The second example will relate the analysis of variability of brain waves in EEG in the frequency domain.

4.4 An Example of Application of CZF method in HRV analysis of R-R time series from ECG

It is well known that R-R time series relate the beat-to-beat time fluctuations of human heartbeat intervals and R-R values are largely controlled by various physiological and psychological factors and, in particular, by the balance between sympathetic and parasympathetic nervous system activity imposed upon the spontaneous discharge frequency of the sinoatrial node.R-R analysis is largely used in psycho-physiological studies. We quote only two papers to outline the importance of such field. The first is an analysis of cardiac signature of emotionality as quoted in ref.24. The second is an analysis of heart period variability and depressive symptoms:gender differences as quoted in ref.25.

Fluctuations in time in R-R result in what we call the variability of the R-R signal and, using the FFT, in the frequency domain three bands are identified. The first, the VLF, is usually considered to range from 0 to 0.04 Hz and related to humoral regulation of the sinus pacemaker cell activity and to other contributing factors; the second, the LF, ranging from 0.04 to 0.15 Hz, and the HF, ranging from 0.15 to 0.4 Hz are roughly correlated to autonomic sympathetic and vagal activities, respectively.

To perform analysis of variability by CZF in the frequency domain we calculate the mean value, $E(R-R)$, in msec. Consequently we will estimate an equivalent frequency

$$f_{equivalent} = \frac{1}{E(R-R)}$$

Finally, we realize the final diagram having on the ordinate the values of the variability as calculated by (4.3) and on the abscissa, in correspondence with each lag , h, we will assign instead the value $hf_{equivalent}$ with $h = 1,2,3,....$

We will now apply the CZF method to the case of the beat-to-beat fluctuations of human hearthbeat intervals in the cases of normal subjects and subjects with pathologies. We will give the CZF results after having performed the analysis of the given R-R time series using also the previously explained other methodologies. We selected four groups of five subjects. Data were taken from Physionet [20].

Let us delineate some features of the experimental data. The first two groups, Y_i and O_i (i=1, 2, 3, 4, 5), are young and old subjects, respectively. Young subjects were (21 – 34) years old and old subjects were (68 – 85) years old. Men and women were included in the two groups All were rigorously-screened and found to be healthy subjects. ECG recording was performed for 120 minutes of continuous supine resting. The continuous ECG, respiration, and (where available) blood pressure signals were digitized at 250 Hz. Each heartbeat was annotated using an automated arrhythmia detection algorithm, and each beat annotation was verified by visual inspection. We selected pieces of 1024 R-R data points corresponding to a time interval of about thirteen minutes. For the other two groups, Vt_i and Vf_i, instead pieces of 1024 data points of R-R time intervals were chosen immediately before the advent of an episode of ventricular tachycardia (V_t) and ventricular fibrillation (V_f).

The first step was to apply the embedding procedure for phase space reconstruction of the given R-R signals. As previously explained, we calculated first the Autocorrelation Function (Au), then the Mean Mutual Information (MI) to select a proper time delay τ . When the results predicted by Au and MI were different, we opted for the time delay as predicted from MI. After this step, we proceeded to the final phase space reconstruction by using the criterion of False Nearest Neighbors (FNN) fixing a threshold value. In order to give some indication, in Figures 1a, 1b, 1c, we give the AutoCorrelation function (Au), MI, and FNN results for some subject of group Yi, In Figures 2a, 2b, 2c, the corresponding results for a subject of group O_i, and, finally, in Figures 3a, 3b, 3c, and Figures 4a, 4b, 4c, those for a subject in group V_{ti} and a subject in group V_{fi}, respectively. All the results are given in Table 1. In spite of different values obtained for Au, it may be seen that rather constant values of time delays were obtained by using MI. They ranged between 1 and 3 for young and old healthy subjects with an embedding dimension that resulted in being constantly equal to 5 for young subjects, and constantly equal to 4 for old subjects. The subjects in V_t gave time delays ranging between 2 and 4 but this time the embedding dimension resulted in varying from 2 to 7. V_f subjects gave time delays between 2 and 4 but the embedding dimension varied from 2 to 8.

Phase space reconstruction resulted in rather homogeneous results in the group of normal subjects, the O_i group, and the Y_i group, with differences in embedding dimension in old subjects (embedding dimension equal to 4) with respect to young subjects (embedding dimension equal to 5). Instead, marked differences arose in the groups V_t and V_f, in the inner of the two groups and with respect to old normal subjects, O_i. Usually, the reconstructed dimension may be indicative of the number of basic variables that are involved in the system under consideration. The obtained results indicate that young subjects show differences with respect to old subjects relative to the number of basic variables involved but such differences are rather moderate. In the case of the two investigated pathologies we are in presence of a very different dynamics and attractor features in the inner of the groups relative to controls. All the arising differences lead to an interpretation in terms of a profound modification and alteration and of a more marked complexity of the dynamics in the V_t and the V_f cases compared to normal subjects. The results indicate that in some cases a larger number of variables while in other cases a smaller number of variables is required. This is indicative of the profound alteration that the two pathologies induce in heart dynamics, compared to the cases of normal subjects.

The second step was to calculate the largest Lyapunov exponent. For brevity, we avoided calculating the whole Lyapunov spectrum. The results are reported in Table 2. All the subjects gave positive values for the exponent. This may be indicative of the presence of chaotic regimes. It may be seen that young subjects gave values trending higher compared to old subjects. Signals immediately before Ventricular Fibrillation gave discordant results in the sense that in one case we had the lowest value of the Lyapunov exponent of the whole experimentation but we had also cases with values very similar to the high values that were obtained in the case of young subjects. On the contrary, signals immediately before Ventricular Tachycardia gave rather low results in two cases. The other remaining values are similar to those previously obtained for the old healthy subjects. In conclusion we had also in this case (as well as in the case of phase space reconstruction) a net variability in the results in the case of pathologies and a rather constant behaviour of λ_E in the case of normal subjects. In our interpretation these results confirmed that the investigated pathologies induce a profound modification and alteration in the dynamics of the two investigated processes compared to normal cases. The statistical results are given also in Table 2.

It is seen that we have significant differences in the case of young subjects vs. old subjects. These are interesting results since, as also outlined in previous papers by other authors [21], this means that the new paradigmatic rule in dynamics of R-R signals is its variability. Young subjects demonstrate a dynamics of R-R intervals that is based on a greater variability compared to old subjects. Age effects on beat-to beat fluctuations in human interbeat intervals involve a progressive reduction of variability and, in accord, we find a statistically significant difference in Largest Lyapunov Exponent between young and old subjects. We find also statistically significant differences in old subjects about thirteen minutes before the advent of an episode of ventricular tachycardia. This is a remarkable result. In fact, it says that we have an index, λ_E , that is able to inform us in advance on the future advent of a so severe an episode in human heart dynamics. Unfortunately, such predictive value is not obtained also in the case of the Ventricular Fibrillation which in fact does not show significant differences compared to the case of old subjects. Other details are given in Table 2.

As third step of our analysis, we must now examine the structure of the investigated signals, and this kind of analysis may be performed by employing the RQA. Let us remember that we calculate a Recurrence Plot and the following variables of interest: the %Rec, the %DET, the %Lam, the T.T., the Entropy, the MaxLine, the Trend. Modifying slightly our previous language, we may reconsider here some of the variables. In particular, the recurrence rate estimates the probability of recurrence of a certain state. Stochastic behaviours cause very short diagonals while deterministic behaviours determine longer diagonals. Consequently, the ratio of recurrence points forming diagonals to all recurrence points, estimates the determinism. Diagonal structures show the range in which a part of the trajectory is rather close to another one at a different time. Therefore, the diagonal length is the time span they will be close and their mean represents the mean prediction time. The inverse of the maximal length line may be interpreted as the maximal positive Lyapunov exponent. The entropy is defined as the Shannon entropy in the histogram of diagonal line lengths. We may also compute the ratio between the recurrence points forming a vertical structures and the whole set of recurrence points. This variable is called Laminarity, and it related to the amount of laminar states and intermittency. In dynamical systems, intermittency is the alternation of phases of apparently periodic and chaotic dynamics. This is useful for the study of transitions (chaos-ordered, or chaos-chaos transitions). (TT), which is the mean length of vertical lines, measures the mean time that the system is trapped in one state or change only very slowly.

This is the basic scheme of RQA. We see that by this set of variables, we may actually explore the inner structure of the given signal, and this is the reason because RQA is so important in analysis of non linear dynamics in signals.

We may now return to consider the specific cases under our investigation. We performed the RQA analysis using a Radius R=20 so to maintain %Rec about 2-4% . This is a methodological attitude that is often usefull in such analysis. We selected a Line L=3, and we used Euclidean distance and mean rescaling. In Figures 5, 6, 7, 8 we give an example of recurrence plot for a subject of Y_i, O_i, V_{ti}, V_{fi}, respectively. The results of the RQA investigation are given in Table 3.

In Table 4 we have instead the statistical analysis of the RQA results.

Before inspection of recurrence plots, we remember the meaning of diagonal lines and in particular the fact that square areas, really a combination of vertical and diagonal lines, indicate laminar areas, intermittency, possibly suggesting transitional regimes as previously discussed. Still, let us observe that in Tables 3 and 4 we introduced a new variable, the Ratio = %Det / %Rec. We see that the signals employed in the investigation have actually a different inner structure. As expected, young subjects give statistically significant different results compared to old subjects for Laminarity, Trapping Time, Entropy, and Max Line. In brief, the two kinds of signals have a very different structure. Statistically significant differences are obtained also in the case of R-R of old subjects compared to R-R of old subjects before the advent of ventricular tachycardia, and this happens for Determinism, Laminarity, Entropy, and MaxLine. This is a remarkable result since by it we are in the condition to anticipate the event. We may predict in advance the advent of ventricular tachycardia. Still, statistically significant differences are obtained in the case of old subjects compared to old subjects with ventricular fibrillation. In this case it is the Ratio variable this is indicative.

Finally, we observe that the structure of the two signals, one before the advent of ventricular tachycardia and the other before the advent of ventricular fibrillation, show significant differences, and this happens for Determinism, and Laminarity. Also this last result is remarkable since it suggests that we have two profoundly different pathologies that may be better studied and understood on the basis of such two variables.

This last observation completes our RQA investigation. In conclusion, we have given a number of important results relating the different structure and the dynamics of the signals under investigation. They all show relevant features that certainly will not fail to be studied and interpreted with care in their proper physiological and clinical context.

Let us conclude with the results obtained by our CZF method. Following the CZF methodology, we calculated the variogram using 1021 lags. On this basis we evaluated the most important parameter of the method, that is, the Total Variability (VT) of the given time series. It was expressed as the square root ot the total variability of the signal obtained for each lag. Therefore the results are expressed in sec. We also calculated the variogram distribution in the frequency domain, in substitution of the classical Fourier transform. Thus, we calculated the variability of R-R in sec^2 in the three bands of interest, VLF, LF, and HF. The results are given in Table 5. In Table 6 we give the results for statistical analysis (t-Test) and in Table 7 those for correlation analysis.

First, let us comment the Total Variability, VT. In the cases under investigation, it shows that young subjects, as expected, show a greater VT compared to old subjects. This parameter increases remarkably in R-R time series before the advent of ventricular fibrillation and of ventricular tachycardia. The statistical analysis reveals that we have a very significant difference in young subjects with respect to old subjects, and, particularly, in old subjects with respect to those with future ventricular fibrillation and in those with future ventricular tachycardia. We may conclude to have found an excellent predictive parameter that is able to anticipate the advent of severe events in hearth dynamics. In addition, we find also that statistically significant differences are maintained for VT in young subjects compared to old subjects for VLF, LF and HF bands in the frequency domain. Still, significant differences are found in old subjects compared to subjects with future ventricular tachycardia for LF and HF bands.

In order to go on in the understanding of such complex phenomena relating pathologies, we have also performed a correlation analysis finding other remarkable results. In young subjects VT results correlated in a significant manner with VLF, LF, and HF. In itself, this result does not appear to be so relevant. It becomes of particular interest when we consider also the results of correlation analysis for old subjects. In fact, in this case we obtain that the total variability of the signals correlates with LF and HF but not with with VLF. This is a very interesting conclusion that deserves to be explained and interpreted in detail under the physiological and clinical profiles. Finally, we obtain still results of particular significance when we apply the correlation analysis to the case of future ventricular fibrillation and of future ventricular tachycardia. In fact, in the case of future ventricular fibrillation we find that correlation maintains between VT and VLF, between VT and LF, and between VT and HF, but in the case of ventricular tachycardia, correlation maintains only between VT and VLF, and between VT and VLF/(LF+HF). These results evidence in a quantitative manner the profound alterations that intervene in health dynamics soon before the advent of ventricular fibrillation and of ventricular tachycardia but also clear in detail the substantial differences that characterize the two pathologies. Certainly, there is here matter for physiologists and clinicians to find a proper understanding and interpretation of such results giving new insights in this matter.

To complete the present section we must still add something about the fractal dynamics of the investigated R-R time series. We previously outlined that a Generalized Fractal Dimension may be calculated by employing the CZF method. Otherwise, and for reason of brevity, we calculated in this section the Hurst exponent. The results are given in Table 8 where is also presented the statistical analysis. Also the analysis of Hurst exponet furnishes relevant results. Using this methodology, it is found that ventricular fibrillation and ventricular tachycardia profoundly modify health dynamics just before of their advent. In fact, by inspection of Tables 8, we see that the values of the Hurst exponent all remain under the value of 0.5, and this result shows that the regime of such R-R time series is of antipersistence and thus of absence of long range correlation. In addition we see that we have statistically significant differences between values in young and old subjects. In addition, very significant differences are found between old subjects and old subjects with future ventricular tachycardia. At the same time very significant differences hold also between old subjects and those with future ventriculat fibrillation. Therefore, we obtain an excellent parameter of prediction of future severe failure in heart dynamics. The reason for such results is that the advent of the mentioned pathologies profoundly alters the fractal structure of the signals taken in consideration in V_t and in V_f. In conclusion, our analysis offers an excellent set of parameters that may be considered as predictive of ventricular tachycardia and ventricular fibrillation.

4.5 The Application of the CZF Method in Analysis of Spontaneous EEG

We called this method as CZKF because its formulation was enriched also by the contributions of another author [17]. We are accustomed to analyse brain patterns of subjects by standard methodologies. Specifically, subjects are instructed to close their eyes and relax. Brain patterns are recorded as wave shapes that commonly show sinusoidal like behaviour. They are measured from peak to peak with a normal ranging from 0.5 to 100 μV. EEG records may be obtained by positioning 21 or more electrodes on the intact scalp and thus recording the changes of the electrical field within the brain. Generally, even up to 128 and more EEG channels can be displayed simultaneously and each corresponding to a standard electrode position on the scalp. The results of EEG signals are usually registered as voltage differences between pairs of electrodes with bipolar leads or between an active electrode and a suitably constructed reference electrode.

The problem in analysing EEG is to provide a proper method to extract its basic quantitative features by accurate procedures. The research regarding the methodology began more than 70 years ago. The basic tool was, and still remains Fourier analysis. The brain states of subjects demonstrate some dominant frequencies; namely:

1) *beta waves (12-30 Hz)*
2) *alpha waves (8-12 Hz)*
3) *theta waves (4-8 Hz)*
4) *delta waves (0.5-4 Hz)*

Over the last two decades the traditional Fourier analysis has been enriched by other methods, including the widespread application of time-frequency methods for signal analysis such as the Wavelet Transform (WT), and the Hilbert transform. These applications have enjoyed varying results. Because of its simplicity, Fourier analysis has dominated and still dominates data analysis efforts. Despite this, as it was outlined in the previous sections, it should be widely recognized that the Fourier transform assumes crucial restrictions which are often violated also in the EEG time series.

The consequences of improper FFT use are significant: the resulting spectrum will make little physical and physiological sense. The brain has an average density of about 10^4 neurons per cubic mm. Neurons are mutually connected into neural nets through synapses. Subjects have about 500 trillion (5×10^{14}) synapses, and the number of synapses per one neuron increases with age while the number of neurons decreases with age. Thus although rather structurally simple, the interconnections produce one of the most massive (functional) structures existing in nature. The natural way to think of this structure is that of a dynamic system governed by laws of non linearity and of non stationarity. We are in presence of a very complex system that again shows a great variedness and variability. Consequently, any method of analysis must quantify these features in order to generate valuable results. To this purpose we propose the CZKF method.

Obviously, the basic feature of the CZKF method is that by it we must estimate the variability that one has in the EEG for each band in a given time interval. This represents the new and important feature of the method. By CZKF we have the opportunity for the first time to evaluate with accuracy the variability of EEG in each of the bands characterizing the brain waves of interest. In this case we will express total variability in microvolts, obviously.

Consider an EEG sampled at 250 Hz. First of all we will calculate the variogram for different lags, h, as previously explained in detail. We will realize a diagram in which we have the values of the variogram in y-axis (ordinate) and correspondingly the $h - lag - values$ on the x-axis (abscissa). Soon after the step will be that one of a conversion of variogram values from time to frequency domain.

We proceed in the following manner:

$$\frac{1}{250Hz} = 0.004\sec$$

In this manner

$$\frac{1}{0.004(lag-h-value=1)}$$

will represent the frequency with the corresponding value of variability at 250 *Hz*.

Similarly,

$$\frac{1}{0.004\times 2(lag-value)}=125Hz$$

will represent the value of variability at 125 *Hz*, and so on for lag values h = 3,4,5,..... . In this manner we may reconstruct the variability of the EEG time series data as a function of the frequency.

Analysis of brain waves will be performed by integration of the calculated variability in each of the four groups of brain waves previously reported summing for each characteristic frequency band. In this manner we will estimate also

$$P(f)=1/f^{\beta}$$

This last discussion completes the exposition of some features of our method. It may be applied to EEG as well as to ERP. In our previous papers [17], we examined eight normal subjects (5 female and 3 male with age ranging from 21 to 28 years old). All the subjects were at rest, watchful but with closed eyes. The sampling frequency was at 250 Hz.

We focused our analysis on the following electrodes: CZ, FZ, O2, and T4. Phase space reconstruction is useless in our case since we had the electrodes positioned on the scalp and their space separation corresponds to time delay. We used the Euclidean Norm that is the time series reconstructed as

$$\sqrt{x_{C_Z}^2(t)+x_{F_Z}^2(t)+x_{O_2}^2(t)+x_{T_4}^2(t)}=X_{EEG}(t)$$

and we calculated the variogram of $X_{EEG}(t)$ at the various lags and subsequently the results were converted into *Hz*. 30000 points of EEG were used, corresponding to 2 minutes of recorded brain activity.

The results are reported in Fig. 9 and in Table 9. It gives an accurate reconstruction of the variability of brain activity in the four bands of interest that are the beta, alpha, theta and delta brain waves. Obviously the method fully substitutes the less appropriate application of FFT, Wavelet, Hilbert transformations and other linear applications.

Finally, we aim to outline here the interest of our CZKF method also in applications in cognitive studies, in analysis of IQ or also, for example, and to evaluate the anesthetic adequacy. In this manner our approach links the previous fundamental studies that are currently conducted by El

Naschie [22] and by Weiss H. and Weiss V [23]. As the CZKF method evidences in detail, the variance of the EEG may be quantified, and is a function of its frequencies. It becomes possible to scale and to measure inter-individual differences – for level of cognition, IQ or anesthetic adequacy not by any absolute score, but by the inter-individual variance of the subjects. Weiss and Weiss [23], in particular, based on empirical data of different authors, showed that thinking can be understood, if we see thoughts as macroscopic ordered (quantum) states in the sense of statistical mechanics. Thinking seems only to be possible, if brain waves use the mathematical properties of the golden ratio and hence of fractal-Cantorian spacetime as discussed by El Naschie [22]. Therefore, a straightforward application of the method and measure here developed is to test the IQ of subjects and correlate the measures arising from CZKF with IQ, using power and variance in the entire range from 3 to about 30 Hz of the EEG.

5. An Analysis of State Anxiety

5.1 Introduction

We will develop now a final application. We will study the state anxiety in humans. We will apply all the previous exposed methodologies. In order to delineate in detail such developed research, we retain that we will help the reading exposing this argument avoiding any possible intermixture with the previous ones, and thus separating this argument from the previous ones, using also references, tables and figures that relate a separate and independent numeration respect to the previous one, used to illustrate the general field of methodologies and applications.

Let us start with a brief discussion on the use of non linear methodologies in psychology.

Psychological data were usually collected in the past psychological studies to assess differences between individuals or groups which were considered to be stable over time (1,2). Instead, a further approach has gained relevance in the past decade, which is aimed to perform an intensive time sampling of psychological variables of individuals or groups at regular intervals, to study time oscillations of the collected data (1). In this way human behavior has been investigated to analyze, for instance, the impact of everyday experience on well-being (3) or the after-effects of negative events (4) or to examine the association between emotions and behavioral settings (5). These studies were often aimed to analyze the nature of rhythmical oscillations in mood and performance of human beings (6). Such an approach leads to progressive changes not only in the methods to sample psychological data but also in our way of thinking about many psychological variables, which may be considered as expression of mind entities unfolding over time (1). A reason to outline the importance of this approach is to acknowledge the role of the human interactions in governing the transitions which continuously take place in mind entities.

It is becoming relevant the notion that our mind, our ideas and convictions are all formed as the results of interactive changes and all they follow possibly a quantum like behavior. Let us explain in detail what we mean by this statement (7,8). For certain questions, individuals have predefined opinions, thoughts, feelings or, still, behaviors. This kind of condition may be considered to be stable in time in the sense that an intensive time sampling of data, consisting as example to questions asked to an individual from an outsider observer or by himself at regular time intervals, will simply record a predefined answer that never will be determined and actualized at the same time the question is posed .In this case, we have a stable dynamic pattern for individuals or groups. The intensive time sampling of data will only confirm an information on time dynamics that is stable in reporting a pattern in self-report or in performance measures with regard to behavior in time of the involved individuals. It has been evidenced (7, 8) that, under the profile of a statistical analysis, the cases as those just mentioned, in which individuals have a predefined opinion or thought that may not be changed in time at the same moment in which questions are actually posed, correspond to a kind of classical dynamics that, statistically speaking, may be analyzed in terms of classical statistical approaches since they are not context dependent (7,8). There are situations in which, instead, a person, who is being questioned by himself or by an outsider observer, has no predefined opinion or thought or feeling or behavior on the given question. The kind of opinion, as example, is formed (that is to say: it is actualized) only at the moment in which the question itself is posed and it is formed on the basis of the context in which the same question is posed. This is a case of a quantum like behavior for a cognitive entity. The core of the difference resides in the fact that in the case of quantum like behavior we are dealing with the actualization of a certain property that is dependent from the instant of time in which the question is posed and thus, in particular, it depends also from the context in which it is posed while, instead, in the classical case all properties are assumed to have a definite connotation before the question itself is posed and thus they are time and context independent. Processes of the first kind are said quantum like, and they follow a quantum like statistics (7, 8). The basic content of such quantum probability approach is the calculation of a probability of actualization of one among different potentialities as result of the individual inspection itself or of an outsider observation. New paradigms are thus emerging in studies regarding mind behavior: one is the concept of potentiality, linked to the concept of actualization. Still, we have the concept of dynamic pattern that is linked to the observation of changing in time as result of the interactive transitions (potentiality-actualization) which take place in human interactions. The case of quantum like behavior is one of the manifold situations in which an intensive time sampling of psychological data, may give important information on the dynamic patterns in self-report and performance measures.

It is noteworthy that people have a defined "sense of self" and accompanying memories of a very early age. It may be due to the fact that the "attractor" of personality (as developed by the brain) has not established a defined enough probability of neuronal connections to establish such a distribution: if neuronal connections are essentially uniform in their shape, it is questionable if an attractor is defined. With repetitive learning inputs, the probability distributions become established (narrowed) and "personality" emerges. Learning skills proceeds along similar lines: repetitive "habits" further narrow the probability distributions so as to make a particular action more refined to the point of not requiring active effort. Both personality and learning, however, are dependent upon the genetics which establish the basic physiology of the neuronal machinery. Predictability regarding personalities and activity is by definition of the singular dynamics, a stochastic process: no matter how narrowed the probability distributions, there always remains a level of uncertainty.

The performance of current neural networks is still too "rigid" in comparison with even simplest biological systems. This rigidity follows from the fact that the behavior of a dynamical system is fully prescribed by initial conditions. The system never "forgets" these conditions: it carries their "burden" all the time. In contrast to this, biological systems are much more flexible: they can forget (if necessary) the past, adapting their behavior to environmental changes.

The thrust here is to discuss the substantially new type of dynamical system for modeling biological behavior introduced as non deterministic dynamics. The approach is motivated by an attempt to remove one of the most fundamental limitations of current models of artificial neural networks—their "rigid" behavior compared to biological systems. As has been previously exposed in detail, the mathematical roots of the rigid behavior of dynamical systems are in the uniqueness of their solutions subject to prescribed initial conditions. Such an uniqueness was very important for modeling energy transformations in mechanical, physical, and chemical systems which have inspired progress in the theory of differential equations. This is why the first concern in the theory of differential equations as well as in dynamical system theory was for the existence of a unique solution provided by so-called Lipschitz conditions. On the contrary, for information processing in brain-style fashion, the uniqueness of solutions for underlying dynamical models becomes a heavy burden which locks up their performance into a single-choice behavior.

A new architecture for neural networks (which model the brain and its processes) is suggested which exploits a novel paradigm in nonlinear dynamics based upon the concept of non-Lipschitz singularities [7, 8]. Due to violations of the Lipschitz conditions at certain critical points, the neural network forgets its past as soon as it approaches these points; the solution at these points branches, and the behavior of the dynamical system becomes unpredictable. Since any vanishingly small input applied at critical points causes a finite response, such an unpredictable system can be controlled by a neurodynamical device which operates by noise and uniquely defines the system behavior by specifying the direction of the motions in the critical points. The super-sensitivity of critical points to external inputs appears to be an important tool for creating chains of coupled subsystems of different scales whose range is theoretically unlimited.

Due to existence of the critical points, the neural network becomes a weakly coupled dynamical system: its neurons (or groups of neurons) are uncoupled (and therefore, can perform parallel tasks) within the periods between the critical points, while the coordination between the independent units (i.e., the collective part of the performance) is carried out at the critical points where the neural network is fully coupled. As a part of the architecture, weakly coupled neural networks acquire the ability to be activated not only by external inputs, but also by internal periodic rhythms. (Such a spontaneous performance resembles brain activity). It must be stressed, however, that behavior may be predicted in the sense of establishing a probability distribution of choices. Thus behavior is not determined, but 'guessed' within the bounds of the probability distribution.

In its most simple form, consider, for example, an equation without uniqueness:
$dx/dt = x^{1/3} \cos \omega t.$
At the singular solution, $x = 0$ (which is unstable, for instance at $t = 0$), a small noise drives the motion to the regular solutions, $x = \pm(2/3\omega \sin \omega t)^{3/2}$ with equal probabilities. Indeed, any prescribed distribution can be implemented by using non-Lipschitz dynamics. It is important to emphasize, however, the fundamental difference between the probabilistic properties of these non-Lipschitz dynamics and those of traditional stochastic or differential equations: the

randomness of stochastic differential equations is caused by random initial conditions, random force or random coefficients; in chaotic equations small (but finite) random changes of initial conditions are amplified by a mechanism of instability. But in both cases the differential operator itself remains deterministic. Thus, there develops a set of "alternating," "deterministic" trajectories.

We would now discuss the reason of a terminology that is delineating. As said, the analogy is with the physics. The state s(t) of a physical entity S at time t represents the reality of this physical entity at that time. In the case of classical physics the state is represented by a point in phase space while in quantum physics it is represented by a unit vector in Hilbert space. In classical terms the state s(t) of the physical entity S determines the values of all the observable quantities connected to S at time t. The state q(t) of a quantum entity is represented instead by a unit vector of Hilbert space, the so called normalized wave function $\psi(r,t)$. For a quantum entity in state $\psi(r,t)$ the values of the observable quantities are potential: this is to say that a quantum entity never has, as example, simultaneously a definite position and a definite momentum and this represents the intrinsic quantum indeterminism that affects reality at this level. We have the relevant concept of potentiality: a quantum entity has the potentiality to realize some definite value for some of its observable quantities. This happens only at the moment of the observation or of measurement and it is this mechanism that realizes a transition from a pure condition of potentiality to a pure condition of actualization. A definite value is not actually realized in the potential state $\psi(r,t)$. A definite values is really actualized only at the moment of the direct observation of some property of the given entity and through the same mechanism of the observation during the act of the measurement. The novel feature is in the transition potentiality → actualization that characterizes the mechanism of observation and measurement.

We have to realize here a large digression in order to clear in detail this point that appears to us of fundamental importance.

As we know all quantum mechanics is based on such binomial conceptualization of potentiality from one hand and actualization from the other hand. In particular, the actualization corresponds to the observation and measurement or, that is to say, to the moment in which we become conscious that some kind of measurement has happened (collapse of wave function) since we read its result by some device. Generally speaking, a system is in a superposition of possible states (superposition principle, potentiality) and such superposition principle is violated in a measurement. This led von Neumann to postulate that we have two fundamentally different types of time evolution for a quantum system. First, there is the casual Schrödinger equation evolution. Second, there is the noncasual change due to a measurement and this second type of evolution (passage from potentiality to actualization) seems incompatible with the Schrödinger form. This situation forced von Neumann to introduce what is usually called the von Neumann postulate of quantum measurement. This happened about 1932. Rather recently, one of us (EC), using two theorems in Clifford algebra, has been able to give a complete justification of von Neumann postulate. The result has appeared on International Journal of Theoretical Physics, and it is available on line [8]. Thus we have given proof of a thing that for eighty years remained a postulate, often discussed and largely questioned. This new result, at least under an algebraic profile, explains the wave function collapse and gives total justification of it, also giving to quantum mechanics an arrangement as self-consistent theory that in the past was often questioned as missing in the theory and signing such missing as a probe of weakness of such theory. In conclusion, the passage potentiality – actualization now seems a more demonstrated transition to which we have to attribute the greatest importance if we do not aim to remain linked to a too limited vision of our reality. On the other hand, there is no matter to continue an infinite discussion on a possible link between quantum mechanics and cognition. We have unequivocal results that demonstrate in detail such point. It is universally accepted that J. von Neumann showed that projection operators represent logical statements. In brief, J. von Neumann showed that we may construct logic starting from quantum mechanics. According to the fundamental papers published by the great logician Yuri Orlov, and in the light of the results that, we repeat, one of us has recently obtained, it may be unequivocally shown that also the inverted passage is possible. Not only we may derive logic on the basis of quantum mechanics. We may derive quantum mechanics from logic. So, the ring is closed. The link between quantum mechanics and cognition is strongly established. The split that occurred between psychology and the physical sciences after the establishment of psychology as an independent discipline cannot continue to encourage a delay in acknowledging this thesis. We may be convinced that there are levels of our reality in which the fundamental features of logic and thus of cognition acquire the same importance as the features of what is being described. Here we no more can separate “matter per se”, in Orlov words, from the features of logic and cognition used to describe it. We lose the possibility of unconditionally defining the truth, as we explained previously, since the definition of truth, now depend on how we observe (and thus we have cognition) the physical reality . Obviously such relativism does not exist in classical mechanics while instead by quantum mechanics we have a Giano picture able to look simultaneously on the left and on the right, at cognitive as well as physical level.

Let us return now to the central problem we have in discussion.

Some mind entities follow quantum like behavior (7, 8). Let us restrict our example to the case of a cognitive entity. A psychological task asks to a participant a question that has a predefined value as answer for each individual. The task asks, as example, to the participant if he (she) has blue eyes. It is clear that the cognitive entity of the participant has a predefined opinion on this question and the measurement, corresponding to the act of posing the question to the subject, will furnish only the trivial recording of an output that is predefined also before the question is posed. There are cases in which the cognitive entity may be submitted to a question for which the person who is being questioned has no opinion ready. He has several potentialities and only one of such potentialities will be actualized at the moment the question is being asked. As example, let us admit that the posed question is the following: are these two geometrical figures equal? (an ambiguous figure). At the moment the question is being asked, the subject has no predefined opinion. He may have, as example, two potential states (possibilities) that are superimposed and they are the two possible answers: yes and no. The cognitive entity will actualize only one answer among the two possible ones at the moment the question is posed and such actualization will correspond to an act of consciousness of the subject. Through the posed question, the subject will be induced to a transition from to a condition of potentiality to that one of actualization. In a quantum like framework, such mechanism of transition from potentiality to actualization will be intrinsically stochastic and strongly dependent from the context in which the cognitive entity of the subject will be induced to answer. Potentiality states of mind entities are superposition of potentialities that are characterized at an ontological level and, as said, among the different potentialities only one state will be actualized corresponding to an act of introspective activity (consciousness advent) of a subject. It is clear that in such cases an intensive time sampling procedure enables to collect data relating subsequent individual acts of introspection, of actualization, of conscious aware and this represents an interesting technique for analysis of mind dynamics.

It is important to outline here that the approach of using an intensive time sampling of psychological data is relevant not only in the cases in which a quantum like behavior may be assumed but, generally speaking, in all the cases of experimentation in which there is the reasonable motivation to retain that it is the dynamic evolution in time of mind entity to cover an important role in the framework of the investigated phenomenology.

It remains to evidence that, through an intensive time sampling of psychological data, we realize a discrete collection of results that usually we call a time series of data. They are actually used extensively in physiological studies of biological signals, and the importance is related to the fact that they contain a fingerprinting of the process under investigation. Consequently, the basic finality of this kind of studies is to analyze the nature of the observed fluctuations in time. Generally, the analysis of the data may enable to establish relevant questions as if time evolution follows a linear or a non linear dynamics, and in particular if it is regulated by deterministic, or chaotic deterministic or noise influenced patterns.

In the present study we investigated the phenomenon of anxiety of state. The finality was to introduce new parameters for the interpretation and control of such psychological manifestation.

5.2 The Phenomenon of Anxiety

Anxiety may represent a proper condition to investigate in detail potentiality of mind entities in analysis of time dynamic pattern. It is well known that fear is profoundly distinguished from anxiety. It is known from many models (9), that fear is a response to a present and actual danger while, generally speaking, anxiety is a response to a potential danger. According to our quantum like model of the previous section, we may say that the anxious individual, at fixed times, may give his conscious introspection and thus evaluating and actualizing a danger that is only potentially fixed. Therefore, fear is a response to a present - real danger, anxiety is instead the response to a potential danger. In various models (10) the risk assessment is seen as the central component of anxiety and it is realized in terms of approaching and scanning potentially dangerous situations. Fear and anxiety can each produce a physiological arousal response that involves activation of the adrenergic system in the CNS and in sympathetic branch of the autonomic nervous system (SNS) (11). Since such identical systems are involved in both such conditions, the phenomenological experience of arousal seems similar, and such similarity of arousal experiences contributes to the common tendency to retain fear and anxiety as either interchangeable manifestations. There are instead substantial differences. The central difference between fear and anxiety should reside in the kind of quantum like behavior that we established in the previous section. The individual in a state of fear perceives the threat that is immediate and real and, on this basis, he gives an active response that in some manner is just induced from the external stimulus. In other terms, the individual actualizes a response that, in some sense, is defined on the basis of the kind of real perceptive stimulus that is offered to him. In the case of anxiety, the individual does not perceive an immediate threat (there is not an external stimulus that actualizes the response). He is focused on a potential threat for the immediate or future times and in many cases he inherites this condition on the basis of his personal history and psychological background (see, as example, the case of a subject with post traumatic stress disorder). In analogy with intrinsic quantum indetermination of physical reality, there is here a proper condition of quantum like uncertainty for mind entity: owing to the indeterministic nature of the anxiety-producing threat, the individual remains suspended into potential states, and usually he cannot determine whether to act or how to act. This is the clear indication of quantum like behavior. The anxiety-producing threat is only potential: the individual feels that there is something that may happen or that might not happen; he remains suspended in a superposition of such potential states. He continues to think about the threat (he remains in the superposition of potential states). He does not react to an attack or to a perception of being attacked, but he remains in the suspended possibility of being attacked. If such individual perceives himself to be really under an attack (actualization), then he will enter an actual fear state.

One very interesting feature is that anxiety represents an emotional condition that is so general and so radical in human that it cannot be considered only a sign of pathology or a defined syndrome but a general mode of the human existence with extreme values that obviously enter in the domain of psychopathology. Therefore, the time analysis of its dynamics offers an excellent opportunity to analyze basic features of a time dynamics regarding in general mind entities of human existence. In addition, while the anxiety of trait may be considered as a rather stable condition of our personality, the anxiety of state is considered more linked to transient phases of our everyday emotional condition, and it may be evaluated by using proper test that were introduced by C. D. Spielberger starting with 1964 (12). It is important to outline that the test may be repeated at fixed time intervals so to have a final time series of collected data that are indicative of the changing in time of the phenomenology under study. The individual is asked to answer to twenty fixed questions that were elaborated (12) with the direct finality to quantify the value of the anxiety of state at the moment of the administered test. For each question, the individual has at his disposal four different modalities of answer with a calibrated score ranging from 1 to 4 according to the seriousness of the emotional condition.

The value of state anxiety for each administered test to the individual, is usually evaluated by direct calculation of the achieved total score and, in case, a statistical analysis may be developed in order to obtain standard statistical indexes over a proper range of time. It is evident that this manner to proceed results to be very limited. We are certainly interested to the values of the test but mainly we must focus our attention on the manner in which variations and oscillations of test values are induced in time from mind entities. For this purpose, the introduction of new methodologies and parameters is really required in order to characterize the dynamic pattern of anxiety of state in individuals: such parameters should be useful also to elaborate diagnostic as well as therapeutic strategies. From the viewpoint of our quantum like model the results of the test must be conceived in the following manner: at fixed times, the individual, through each posed question, exerts an introspective activity on himself (an act of conscious awareness): by each introspective act the subject makes a transition from a superposition of four potential states (the four kinds of answer that are at his disposal) to the final actualization of only one among such four potentialities. None of the four potentialities is predefined previously the question is posed (superposition of potential states) and only one among the different potentialities is actualized only at the moment of the conscious introspection (transition potentiality-actualization). Obviously, there exists also here a limit in our experimentation. When the subject repeat his(her) test for the second time, he just knows what question will be posed to him and this situation could influence his answer. However, we will admit here that the subject, a control subject not affected from pathologies, will be able to answer to the posed questions without suffering a strong conditioning arising from the fact that he previously knows the posed questions. Our aim is to investigate the nature of such transitions, potentiality-actualization, in time.

5.3 Materials and Methods

Six healthy subjects were examined: F. Dav., male, 30 years old, D.Pet., female, 25 years old, A.Mac., female, 55 years old, G.Den., female, 30 years old, A.Men., male, 57 years old, M. Den., female, 32 years old. Each subject was subjected to the test four times in one day and precisely at each time step of three hours starting with the waking up. The collection of data proceeded for about 30 days. Time series data were collected by the test given to each subject . The resulting time series data for the subject D.Pet. is reported in Fig.1 to give an example of the obtained experimental time dynamic pattern.

5.4 Results of Poincaré-plot Analysis of the Data

In this section, we aim to introduce new indexes that in our opinion may help in the characterization of the investigated process.

As usual, our examination of the data started with the elaboration of a statistical analysis for the six examined subjects. The results are reported in Tables 1-6 for each subject. Mainly, we calculated the mean, the standard deviation and the variance of the scores obtained in about 30 days. Of importance it must be considered the value of the variance since, as previously said, we were mainly interested to investigate the phenomenology of the variations, and thus of the oscillations and of the fluctuations of the test value during the time period of its administration. We added also some other statistical indexes as the Median, the Minimum-Maximum values, the Root Mean Squared, the Skewness and the Kurtosis to have a clear characterization of the correctness of our samples under a statistical profile. In fact, it may be verified by these indexes that all the subjects responded to the test with full adherence to the requirements of the correct statistical samples.

A subject reached a mean value of 23.4 for the test, the other reached 30.3, the subsequent obtained 38.1, the other had 38.3 and, finally, the two remaining subjects had 47.2 and 53.4 respectively. It is important to outline here that the test furnishes usually four different scales for the evaluation of the score, the first with score value of 20 (very moderate level of anxiety of state), the second with value ranging from 21 to 40 (moderate level of anxiety of state), the third with score value ranging from 41 to 60 (high level of anxiety of state), and the fourth with score value ranging from 61 to 80 (very high level of anxiety of state). Therefore, four subjects resulted to have a moderate level of anxiety of state, and the remaining two subjects resulted instead to have an high level of anxiety of state. Note that the use of the mean value of the test in time and the subdivision of the test score in four intervals does not help for a correct diagnostic identification of the dynamic pattern of each subject in time. Looking at the values of the

Standard Deviations and of the Variances for such subjects, one catches sight of profound differences among subjects included instead in the same interval. As example, in the case of two subjects we have mean values of 38.1 and 38.3 that are very similar under the profile of the mean score but they exhibit profound differences under the profile of their variability in time since one has a Standard Deviation of 6.39 and a Variance of 40.86 while the other subjects has a Standard Deviation of 9.99, and a Variance of 99.89. Therefore, it derives that in no manner the mean value of score in the test for anxiety of state may be assumed to represent the correct diagnostic profile of the anxiety of state of a subject in time. From the previous section we know that such dynamic profile could be quantum like and as such marked from pure stochastic behaviors. Therefore we must be interested to a very deep analysis of such time variations, as oscillations and fluctuations of state anxiety in time and to this purpose the introduction of proper new indexes is primarily required. Before of all, in order to proceed along an accurate characterization of the time dynamic of state anxiety of subjects, we aim to introduce two new indexes. They are obtained reconstructing a kind of phase space with x_i values in abscissa against x_{i+1} values in ordinate. On a fitted ellipse we identify two indexes, the first, that we call here SD1 in analogy with previous studies on heart rate variability, expresses the tendency to the variability of the score for each subject in the short time intervals, and the second, that we call SD2 for the same analogy, expresses instead the tendency to the variability in the score along a consistent time interval. The use of such both indexes, SD1 and SD2, gives us the manner to characterize and to examine the dynamic tendency of state anxiety for each subject along the time interval of the investigation, considering variations of this phenomenon in the brief interval of time as well as in the larger time interval.

Poincaré-plots (13) are currently employed to investigate the complex dynamics of non linear processes as those given in Fig.1. A two-dimensional phase space may be used to visualize the information contained in a given time series. In a Cartesian co-ordinate system a point P_i is defined by the time interval T_i and τ intervals subsequently following T_i, thus giving $P_i(T_i,T_{i+\tau})$, being τ a proper time delay that in studies of chaotic-deterministic time series may be estimated by using Autocorrelation Function and Mutual Information Function (14). In this our preliminary investigation a time delay τ=1 was selected by us. In this manner, the Poincaré-plot resulted to be a diagram in which each data of the given time series is plotted as a function of the previous one (τ=1), this plot gives a visual inspection of the given time series data by representing qualitatively with graphic means the kind of variations of such data fingerprinted during their collection. The realized plots may be analyzed also quantitatively. This quantitative method of analysis is based on the assumption of different temporal effects of changes on the subsequent time series data without a requirement for a stationary behavior of data itself. Analysis, generally, entails fitting an ellipse to the plot with its center coinciding with the center point of the markings. The line defined as axis 2 shows the slope of the longitudinal axis, whereas axis 1 defines the transverse slope that is perpendicular to axis 2. Usually, the Poincaré-plot is first round 45 degree ring, clockwise. The standard deviation of the plot data is then computed around the axis 2 and passing through the data center. The first index, SD1, is so calculated. SD1 accounts for the variability of the data for short intervals of time. The standard deviation of long term data is quantified by turning the plot 45 degree ring, counterclockwise, and by computing this time for data points around axis 1 which passes through the center of the data. SD2 is calculated and it accounts for variability of data for long term time intervals. In conclusion, given the time series data, we may introduce two indexes, SD1 and SD2 respectively, that account for the variability of the analyzed data in short as well as long intervals of time. Applying this kind of analysis to our time series of data, we become able to estimate how is expected variability of state anxiety in subjects in short as well as in long intervals of times. The particular relevance of such two introduced indexes must be thus clear. It seems reasonable to conclude that subjects with low values of SD1 and SD2 will exhibit low levels of state anxiety while from a psychological and clinical viewpoint it will be carefully characterized the condition of subjects with high values of SD1 and SD2 or of SD1 and not SD2 or viceversa. A new phenomenology of state anxiety is so delineated, and it will be characterized by levels of state anxiety that will result to be discriminated and carefully characterized respect to the proper case of normality (low level of state anxiety). Statistically speaking, the plot will display the correlation possibly existing between consecutive data (scores of the test) in a graphical manner. Non linear dynamics considers the Poincaré-plot as a two dimensional reconstructed time series data phase space which is a projection of the reconstructed attractor describing, in our case, the dynamic of the mind entities responsible for state anxiety.

Concluding: The time series data of state anxiety will give a Poincaré-plot that typically will appear as an elongated cloud of points oriented along the line of identity. The dispersion of points perpendicular to the line of identity will reflect the level of short term variability of the score for state anxiety (SD1) while the dispersion of points along the line of identity will indicate the level of long term variability of the score for state anxiety (SD2). The elliptic structure will mirror instead the basic periodicity of the data and thus, as an important indication, it will correspond to the possible periodicity of the scores during the administered test.

We performed this analysis for the six subjects. The results of the Poincaré-plots are reported in Figures 2-7 while the quantitative results for SD1 and SD2, respectively, are given in Table 7 where they are compared with the mean values (s.d. and variances)of the scores of the test as they were previously calculated. The satisfactory predictive power of SD1 and SD2 is clearly evidenced.

Let us comment briefly some results. The subject F. Dav reported a mean value of 23.4 with st. dev. of 2.7 and a variance of 8.30. SD1 resulted 2.11 while instead SD2 reached the value of 3.74. This means that in the short time interval such subject varied his score of only 2.11(thus ranging in mean from 21.29 to 25.51) and thus remaining any way in a moderate level of state anxiety. In the long intervals of time his score changed of 3.74 and thus ranging in mean from 20.00 to 27.14 that is still low and very similar to variability in short time intervals. In conclusion this subject had a rather stable condition of moderate state anxiety.

The subject A. Men had a mean value of 30.3 with a st. dev. of 2.4 and a variance of 7.36. It resulted SD1=2.21 and SD2=3.13. The time dynamic of state anxiety of this subject seems to be very similar to that one of the previous subject with a rather stable tendency to remain in the condition of moderate level of anxiety in short as well in long intervals of times.

Let us consider now the case of A. Mac. who had a mean value of 38.1 with a st. dev. of 6.39 and a variance of 40.86. From the statistical data we deduce that his mean value is only of 7.8 points greater than A. Men. (corresponding to about 21%) but standard deviations and variances result to be very different. Owing to the great value of the variance we expect for such subject a great tendency to variability and it is of importance to establish if such tendency to time variability regards the short or the long time intervals or both. The use of Poincaré-plot gives this kind of information. In fact, SD1 resulted to be 5.46 while instead SD2 gave the value of 7.33. Comparing such results with those of F. Dav and of A. Men., we conclude that the subject A. Mac. has a tendency to a great variability both in short as well in long time intervals. In short times his score may vary in mean ranging from 32.64 to 43.56 (he may reach also the high level of state anxiety), and in the long interval of time, his score may vary in mean from 30.80 to 45.40 (he may reach the high level of state anxiety also greater than ones of short time terms). In conclusion, this subjects has dynamic features that result to be very different from the previous ones. As the first two subjects, A. Mac. starts in mean with a moderate level of state anxiety but in the short time intervals as well as in the long time intervals he has the tendency to reach also high levels of state anxiety.

In conclusion, as seen, SD1 and SD2 compete in an evident manner to differentiate in detail the dynamic of state anxiety also for subjects that have very similar scores.

Let us examine now a very different situation. The subject D. Pet had a mean value of score of 38.3 with a st. dev. of 9.00 and a variance of 99.89. Note that really the mean value (38.3) of this subject is substantially the same (38.1) of the previous subject A. Mac. Profound differences exist instead for st. dev. and variances indicating that, in spite of very similar results for the test, the two subjects exhibited very different dynamic patterns that are important to characterize. In fact, calculating SD1 and SD2, we obtain that SD1=5.82 while SD2 actually assumes the value of 12.96. In comparison with A. Mac, the subject D. Pet. has a very similar value of SD1 (5.82 vs 5.46) but a very large difference for SD2 (12.96 vs 7.33). In the long time intervals this subjects presents a variability that may be also of about 13 times higher, confining him in a condition of high state anxiety. Therefore his dynamic pattern is very different from that one of A. Mac although the scores of the test resulted substantially the same (38.3 vs 38.1).

In addition the value of SD1 and SD2 for D. Pet. may be compared with the previous ones of F. Dav. that showed the most stable condition of moderate state anxiety . In this case, we may evidence a great suitability of SD1 (5.82 vs 2.11) and SD2 (12.96 vs 3.74) to actually characterize state anxiety of subjects and their variability in time. Looking at the results of Table 7 we may still comment the values of SD1 and SD2 that were obtained for the remaining subjects, G. Den and M. Den, observing that such indexes still continue to characterize in detail the time variability of state anxiety also for such subject.

In conclusion, we suggest that, in addition to the scores that are collected by the test, two other indexes should be adopted in order to proper characterize time variability and thus time dynamics of state anxiety of individuals and they are SD1 and SD2 as they are obtained by analysis of the obtained time series data by using Poincaré-plots.

5.5 Results of Variogram and Fractal Analysis

If SD1 and SD2 are two quantitative indexes that, as seen, are suitable to characterize the variability in short as well as in long time intervals for time series data regarding state anxiety, such indexes, of course, cannot give any detailed information on the time dynamics that characterizes state anxiety. In order to reach this objective, a kind of non linear analysis must be still performed using some other elaborate techniques.

Let us start considering the notion of fractal. This term was introduced (15) in 1983 by B.B. Mandelbrot. A fractal object is made of parts that are similar to the whole in some way, either the same except for scale or statistically the same. The chaos dynamic mechanism and the interaction of non linear processes may be an essential cause of uneven distributions of data which results in fractal structure. Self-similarity or statistical self-similarity may be investigated in given time series data with the finality to establish their fractal or multifractal behavior. A formal definition of a self-similar fractal in a two-dimensional x-y-space is that f(rx, ry) is statistically similar to f(x, y) where r is a scaling factor. This may be quantified by applications of the fractal relation

$$N = C\, r^{-D} \qquad (5.1)$$

where r is a characteristic linear dimension, D is the fractal dimension (real number >0), C is a constant of proportionality, the pre-factor parameter, N=N(>r) is the number of objects with characteristic linear dimension ≥ r .

As example, the number of boxes with dimension x_1 and y_1 required to cover a given object is N_1 and the number of boxes with dimensions $x_2 = r\, x_1$, $y_2 = r\, y_1$ required to cover the object is N_2. If the object is a self-similar fractal, we have that

$$N_2/N_1=r^{-D}$$

In the same manner one may consider a self-similar fractal in a n-dimensional $x_1, x_2, \ldots, x_n$-space with $f(rx_1,rx_2,\ldots,rx_n)$ statistically similar to $f(x_1,x_2,\ldots,x_n)$. with r scaling factor.

Many physiological processes posses scale similarity (scale-invariance) properties. Self-similarity or statistical self-similarity may be investigated in given time series data with the finality to establish their fractal or multi fractal behavior (16). In the present paper we will adopt the following simple procedure.

Let us take now the notion of variogram previously exposed.

Consider the importance to have introduced here an analysis by variogram of time series regarding state anxiety. While the previously introduced indexes SD1 and SD2 give a general indication on variability in time of state anxiety in short as well as in long time intervals, variogram enables to quantify such time variability at each lag time. In particular, a small value of the variogram will indicate that pairs of results of the given time series are similar or have a low variability at a particular time distance of separation. Of course, high values of the variogram will indicate instead that the values are very dissimilar or that we have high variability.
The results of the variograms are reported in Figures 8-13 while the results of the fractal analysis are reported in Table 8.

The analysis of the variograms reveals some important features. The subjects F. Dav and A. Men gave the most modest values of variograms ranging from 0 to 8.34 at least. They had a low variability in time. This result is in accord with the mean value of the test that in fact gave the lowest values for such two subjects. Also the statistical values of st. dev. and of variance resulted very contained. Corresponding such subjects gave also the lowest values for SD1 and SD2 respectively. Note also that the variogram showed the tendency to decrease progressively (decreasing variability) after 20 lags (about 5 days) and to annul itself in about 80-90 lags corresponding to about 500 hours. This behavior reveals the tendency of state anxiety in such two subjects to vary with some periodicity concluding its cycle in about twenty days. Of course the tendency of the variogram to a progressive decrease (progressively decreasing variability) resulted mixed to time lag intervals with variogram showing increased variability as example, at 30, 50, 70 time lags corresponding to 180, 300, and 420 hours.

Soon after, the subject A. Mac showed a more marked variability with a variogram ranging from 0 to 43.51. It is important to outline that also in this case we have an excellent agreement with the mean value of the test, the statistical indexes and the values of SD1 and SD2 respectively. Also in this case the variogram showed the tendency to decrease progressively its variability and to annul itself in about 80 lags. Again it followed an initial increase until 20 lags and still we had mixed time lag intervals of increasing variability at about 30, 50, 70 time lags.

The same important results are obtained by inspection of the variogram regarding the subject G. Den. In this case the score of the test was of 53.4 in mean with a st. dev. of 8.8 and variance of 82.67. Correspondingly, the variogram also increased its maximum value ranging this time from 0 to 84.95. Also Sd1 and SD2 increased their values. The behavior of the variogram remained the same as in the previous cases, differing only for the assumed values. In particular, it increased until a time lag of about 20 lags and thus it decreased progressively and annulled itself in about 80-90 lags with mixed peaks at about 30, 50, 70 lags. In substance, they were the values of the variogram to differentiate the behavior of this subject respect to the other subjects while apparently the time dynamics remained unvaried for all the examined subjects. The same conclusions may be reached examining the case of the subject M. Den. This time the mean value of the test reached 47.2 with a st. dev. of 10.3 and a variance of 119.77. Respect to the subject G. Den, the score of the test resulted in mean lightly less (47.2 vs 53.4) but the st. dev. (10.3 vs 8.8) and the variances resulted greater (119.77 vs 82.67). The corresponding variogram assumed still an higher value ranging this time from 0 to 130.9. In correspondence also SD1 and SD2 resulted strongly increased and, in detail, SD1 resulted to be 4.72 and SD2 assumed the value of 10.3.

The time lag behavior resulted the same as in the previous cases with the exception, obviously, of the assumed values. Also this time it increased until about 20 lags and thus it started to decrease annulling itself about 80-90 lags. Peaks were found again about 30, 50, 70 lags.

The subject D. Pet showed instead some important differences. He had a mean score of the test of 38.3 (very similar to the score 38.1 of the subject A. Mac), but he had a st.dev. of 9.00 and a variance of 99.89, an high value in the investigated group. In correspondence SD1 resulted to be 58.2 but SD2 resulted to be 12.96, the highest value in the group of subjects. In conclusion, in spite of a moderate value of his score (38.3 mean value), this subject showed the highest variability in time dynamics of his state anxiety. In correspondence, the variogram resulted ranging from 0 to 100 and the time lag dynamics showed some modifications respect to the case of the other subjects. As in the previous cases, it increased until 20 lags and than it started to decrease but annulling itself, this time about 110-120 lags. Still, very marked peaks this time appeared at about 20, 40, 60, 80-90 lags. In brief, we had some modifications in time dynamics but especially in the time variability of the dynamics of this subject.

The explanation may be found analyzing the results of fractal analysis that are given in Table 8. Before of all, we have to outline that our analysis indicates for the first time that state anxiety responds to a fractal structure. We have a kind of multiplicative process possibly supported by additive noise. The Fractal Measure more than the Generalized Fractal Dimension reveals that we had moderate values of such parameter in correspondence of low values for mean score of the test, of st.dev., of variance, of SD1 and SD2 while instead we had progressively increasing values of Fractal Measure for increasing values of the mean value of score, of st.dev., of variance and of SD1 and SD2 in the case of the other subjects. In spite of a rather stable value for Generalized Fractal Dimension, we had value for Fractal Measure that progressively range from 13.2 to 305.00 with a net differentiation and thus a discriminating ability.

5.6 Linear Analysis in Frequency Domain

In order to deepen the results that we obtained about the recurrent components that we identified by variogram analysis, we performed a further preliminary analysis calculating Fourier spectrum of the time series data of the six examined subjects. We must remember here that the limit of this kind of analysis is that it is a linear method in the framework of a process that instead is intrinsically non linear. However, we arrived to obtain some preliminary interesting information. In frequency domain we calculated an AR spectrum by using an AR model at order 16. We evidence for the first time that time behavior of state anxiety exhibits some harmonic components peaked at some specific frequencies that we identified in all the examined subjects. The basic features of such spectra are summarized in Figures 14-19 and in Table 9. We identified four bands of interest. The first about 0.1 Hz, the second about 0.2 Hz, the third in the region 0.3-0.4 Hz, the fourth about 0.5 Hz. Note that in ordinate we have always the test score as reference. The actual value is obtained by square root of power spectrum and multiplying by 100.

As we know, we sampled the time series of subjects at time steps of about three hours. The frequency value was of $9.25x10^{-5}$ Hz. The spectra are given, in accordance with the Nyquist theorem, at 0.5 of such value. By such analysis it is seen that four peaks are always present in all the examined spectra. All they are given at the following times. One period of time is about 30 hours, the other is given about 15 hours, the third about 7.5-10 hours and, finally, the last about 6 hours. We find for the first time that state anxiety runs again quite periodically with times of 30,15,7.5-10, 6.0 hours. This is a very important result also for diagnostic and therapeutic reasons.

Conclusions

We started admitting a quantum like model for behavior of mind entities in state anxiety of human subjects. Our aim was to investigate time dynamic variability of data preparing several experimental time series that were obtained by using the well known test of D. Spielberger as it was arranged starting with 1964. We obtained that the dynamic of this process follows a fractal regime possibly a quantum fractal behavior (18). In order to proceed with a quantification of the basic features of the time series under investigation, in addition to fractal analysis, we introduced several parameters, and, in detail, the Poincaré-plots with linked the indexes SD1 and SD2, quantifying time variability of the data along short as well as long times, and an analysis of time series data by a variograms. We found that SD1 and SD2 are very satisfactory indexes that may be used to characterize in detail time variability of state anxiety in human subjects. Also analysis by variograms confirmed its predictive attitude. It resulted able to delineate time variability of state anxiety at each time step and to differentiate among the different conditions of variability of human subjects. In particular, the use of variogram analysis enabled us also to identify an important feature of dynamics of the engaged mind entities. We found that the variograms of the different six subjects exhibit some constant recurrences in lags and thus in time: the variograms assumed always the same increasing and decreasing behavior at about the same times with pronounced peaks of variability still at the same recurrent times and finally such variograms annulling themselves also at recurrent times. This result suggests that the engaged mind entities behave in time following a proper inner function. In fact, the variograms of different subjects presented the same kinds of recurrences in time for all the subjects, also submitted to different environmental conditions. In conclusion, the state anxiety seems to represent an emotional human condition that is so general and so radical in human to express a common mode of human existence in time, regulated in the inner of mind entities by the same recurrent, deterministic like, function. In particular it was estimated by us that such recurrent mind function seems to repeat itself with periodicity like of about twenty days and giving again basic features of self-similarity. This recurrent function results instead to be differentiated in subjects, from subject to subject, only for the different values that it assumes at the same prefixed times. In conclusion, the state anxiety shows a rather constant tendency to be recurrent in time with an inner deterministic - periodic like mechanism. Harmonic components were also found when we submitted time series data to frequency domain analysis by FFT.

Finally, there are some other important questions that we examined in the present paper. We attributed a great importance to the analysis of the time variability of the data of time series that were investigated.. The different scores that were obtained in mean for the test of the six subjects, linked to the different values of st.dev. and variances, SD1 and SD2, and compared with the results of fractal analysis, indicated that the increasing mean values of the score of the test, of st. dev., of variance and of SD1 and SD2 correspond to an increasing time variability of the data in a recurrent functional framework that of course remained instead rather constant for all the six subjects not in the assumed values but in the temporal behavior. As general representation of the process, it seems thus emerging a framework in which we have a basic recurrent, deterministic like, process whose time behavior remains rather similar for all the six subjects but it is, instead, differentiated from subjects to subject owing to the variations and variability in the values that in time such basic function assumes in correspondence of the different states of anxiety characterizing the different subjects. This seems to represent an interesting result that we would comment in more detail.

As previously we said, state anxiety represents an emotional condition that is so general and so radical in human that it cannot be considered only a sign of pathology or a defined syndrome but a general mode of the human existence. In our opinion, it represents consequently a proper condition to investigate on a general plane some features of mind entities in analysis of their time dynamic pattern. In state anxiety, anxiety is activated uniquely in the conditions in which the subject evaluates his living situations as a threat and consequently he activates a sequence of behaviors as generally they are induced from anxiety. We have suggested a quantum like model for this process assuming a superposition of potential states in mind entities before the subject actives introspection. In detail our model runs as it follows.

1- State anxiety rises on the basis of inner motivations of the subject.

This is to say that an inner stimulus as thoughts, feelings, biological needs,…is configured in mind entities as a superposition of potentialities. To give an example, remaining on the general plane let us examine the kind of emotional response that a subject could give to an event. In the case of a quantum like superposition of potentialities , we will have the following indicative expression

$$\psi = c_1|\text{frustrated}> + c_2|\text{anxious}> + c_3|\text{excited}> + c_4|\text{angry}> + \ldots\ldots\ldots \quad (5.2)$$

where ψ will represent the whole potential state of the mind entity of the subject for the emotion response . Each |… > will represent each potential state of the emotion response dynamics and the complex numbers c_i (i = 1,2,…..) will be probability amplitudes so that $|c_i|^2$ will represent the probability that the potential state i will be actually recognized (actualized) at cognitive level when the subject will actualize his response thinking about his situation.

2- At the level of state anxiety, the initial stimulus will be inner (thoughts, feeling, …..) and, still again, we will have a superposition of potentialities as response. As example, with regard to the possibility for the subject to feel excitement as consequence of such inner stimulus, we will have

$$\psi = c_1|\text{ very moderate excitement }> + c_2|\text{ moderate excitement }> + c_3|\text{ quite high excitement }> + c_4|\text{ very high excitement }> \quad (5.3)$$

This is the superposition of potentialities at the level of mind entities.

3-The following step is that the subject will perform a cognitive evaluation. He will perform an introspective activity, an act of consciousness, and by this act, he will give actualization to only one among the various potentialities before mentioned in the (5.2 or 5.3). He will perform a transition potentiality → actualization giving to himself to be in the actual state 1 or 2 or 3 or 4. The first actualization will be performed with probability $|c_1|^2$, the second with probability $|c_2|^2$, the third with probability $|c_3|^2$ and the fourth with probability $|c_4|^2$. One actualization among the different possibilities will give also a score as result of the answer given from the subject submitted to the various questions posed by the test. The same mechanism will happen for the other posed questions of the test.

Note that the particular importance of the (5.2 or 5.3) resides in the term superposition that we have employed for it. The potential states (5.2 or 5.3) represents the simultaneous presence of the four potentialities in mind entities of the subject. Consequently, the deriving model is that one of an intrinsic indetermination for mind entity at this stage. Such intrinsic and ontological indetermination is released only at the moment of the individual cognitive evaluation when he actualizes one and only one of the possibilities at his disposal and cancels the previous indetermination.

4- As consequence of the actualization during the cognitive evaluation, the subject will experience a number of subjective feelings, of apprehensions, of anxious expectations also with activation (arousal) of his nervous system, and with the final evidence of some subjective behaviors.

5-Some control and /or defence mechanisms will interfere with this dynamic. They will have the finality to give adaptability to the subject and reduction of anxiety.

The time series data that we collected for the six subjects reflect in some manner all this time dynamics, and we must expect that, in correspondence to the different mean values that were obtained as result of the test, the different subjects characterized the different levels of indetermination that, as previously seen, represent the crucial point of the whole process generating state anxiety.

Let us give still some examples in order to be clear. For a subject with a very moderate or moderate mean value of the test of state anxiety we should have that the values of probabilities $|c_1|^2$ and $|c_2|^2$ of the (5.2 or 5.3) , just corresponding to a moderate anxiety, will be very high while there will be present very low values of probabilities of $|c_3|^2$ and $|c_4|^2$, corresponding instead to high anxiety. We will have approximately that

$$|c_1|^2 + |c_2|^2 \equiv 1 \quad \text{and} \quad |c_3|^2 \rightarrow 0 \text{ and } |c_4|^2 \rightarrow 0 \tag{5.4}$$

This will be true for all the questions posed to the subject during the test.

A subject having a very moderate or a moderate anxiety will be suspended really between two potential states (1) and (2) instead of (1), (2), (3), (4), being $|c_3|^2 \equiv 0$ and $|c_4|^2 \equiv 0$ and thus he will have a more moderate indetermination respect to the general case.

The subjects with an high mean value of the score and an high variability in time, will have

$$|c_1|^2 + |c_2|^2 + |c_3|^2 + |c_4|^2 \equiv 1 \quad \text{with} \quad |c_3|^2 + |c_4|^2 > |c_1|^2 + |c_2|^2 \tag{5.5}$$

with all the four potential states having the concrete possibility of being actualized and thus such subjects will show greater indetermination and greater variability of data respect to the previous case.

We may say that in the first case we have a lower indetermination in the potential states respect to the second one. This is to say that the subjects having higher mean score should exhibit more elevate indetermination respect to the case of subjects with less mean score. Obviously, in the case of more elevate values for scores and thus of indetermination, we expect that more hardly control mechanisms acted to reduce state anxiety or to induce adaptability in the investigated subjects, and such systematic action of mind and biological control induced high variability in the measured data. This is the reason because we found so marked differences in st. dev., in variances, in SD1 and SD2, and in variograms in the different examined subjects. The found time variability of data was also direct expressions of the acting mechanism of mental and biological control and defence, and this was, in conclusion, the reason because, from its starting, we attributed so much attention to our analysis of time variability of data. They are indications of the great indetermination that is at the basis of this process as well as of the basic mechanisms of control that consequently enter in action. They, of course, represent the central core of the mechanisms to be understood in analysis of state anxiety. It is this reason because the quantitative indexes, that we introduced, seem to be of relevant importance. They are just able to characterize and to quantify indeterminism and acting control mechanisms in the dynamics of state anxiety of subjects.

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Table 1 Embedding Analysis

Subjects	Autocorrelation Function (Au)	Mutual Information (MI)	False Nearest Neighbors (FNN)
Y_1	10	3	5
Y_2	103	1	5
Y_3	32	3	5
Y_4	17	2	5
Y_5	19	2	5
O_1	12	1	4
O_2	385	2	4
O_3	110	2	4
O_5	16	3	4
O_6	23	3	4
Vt_3-13	312	2	4
Vt_2-67	86	4	4
Vt_1-26	32	3	7
Vt_1-15	357	2	2
Vt_1-03	139	2	6
Vf_2-30	21	4	8
Vf_2-71	1	4	5
Vf_1-8013	90	4	4
Vf_1-217	1	2	4
Vf_1-115	358	3	2

Table 2 Largest Lyapunov Exponent

Subjects	λ_E		Statistical analysis (t-Test)	
Y_1	0.625 ± 0.054		**Y_i vs O_i**	
Y_2	0.635 ± 0.052		P value	0.0066
Y_3	0.645 ± 0.055		P value summary	**
Y_4	0.625 ± 0.049		Are means signif. different? ($P < 0.05$)	Yes
Y_5	0.521 ± 0.053		t, df	t=3.634 df=8
O_1	0.562 ± 0.047		**O_i vs Vt_i**	
O_2	0.440 ± .044		P value	0.0281
O_3	0.523 ± 0.052		P value summary	*

O_5	0.439 ± 0.055		Are means signif. different? (P < 0.05)	Yes
O_6	0.490 ± .066		t, df	t=2.675 df=8
Vt_3-13	0.373 ± 0.063		**O_i vs Vf_i**	
Vt_2-67	0.432 ± 0.085		P value	0.787
Vt_1-26	0.430 ± 0.094		P value summary	ns
Vt_1-15	0.150 ± 0.058		Are means signif. different? (P < 0.05)	No
Vt_1-03	0.294 ± 0.113		t, df	t=0.2794 df=8
Vf_2-30	0.498 ± 0.122		**Y_i vs Vt_i**	
Vf_2-71	0.605 ± 0.083		P value	0.0014
Vf_1-8013	0.648 ± 0.074		P value summary	**
Vf_1-217	0.668 ± 0.066		Are means signif. different? (P < 0.05)	Yes
Vf_1-115	0.168 ± 0.098		t, df	t=4.777 df=8
			Y_i vs Vf_i	
			P value	0.3567
			P value summary	ns
			Are means signif. different? (P < 0.05)	No
			t, df	t=0.9780 df=8
			Vt_i vs Vf_i	
			P value	0.1257
			P value summary	ns
			Are means signif. different? (P < 0.05)	No
			t, df	t=1.710 df=8

Table 3 RQA Analysis

Subjects	**% Rec**	**% Det**	**% Lam**	**T.T.**	**Ratio**	**Entropy**	**Max Line**	**Trend**
Y_1	0.171	0.342	0.685	3.000	2.000	0.000	3	0.090
Y_2	0.391	33.842	0.148	3.000	86.638	1.491	8	-0.252
Y_3	0.132	1.037	0.000	0.000	7.859	0.000	7	-0.216
Y_4	0.161	0.481	0.000	0.000	2.979	0.000	4	-0.079
Y_5	0.369	9.716	0.158	3.000	26.313	1.777	8	-0.344
O_1	3.011	53.349	24.458	4.181	17.721	2.247	22	-2.743
O_2	2.941	16.617	15.304	3.807	5.650	2.435	18	-7.875

O_3	1.027	10.087	6.927	3.773	9.824	1.984	16	-1.119
O_5	1.099	11.384	19.940	3.850	10.356	2.524	15	-0.744
O_6	1.389	21.329	33.343	4.089	15.351	2.835	30	-1.452
Vt_3-13	9.354	81.594	88.203	9.482	8.723	4.086	185	-21.035
Vt_2-67	6.137	72.819	78.434	11.743	11.865	4.140	85	-1.163
Vt_1-26	3.294	63.040	75.824	6.100	19.136	3.783	99	-3.408
Vt_1-15	22.430	94.955	96.215	42.409	4.233	6.057	617	-70.089
Vt_1-03	12.528	87.955	91.904	15.395	7.021	4.632	281	-20.9
Vf_2-30	3.756	37.225	55.763	15.284	9.911	3.481	94	-7.896
Vf_2-71	0.371	2.446	0.159	3.000	6.598	1.677	9	0.092
Vf_1-8013	1.173	6.098	8.214	3.852	5.197	1.476	12	0.117
Vf_1-217	20.904	15.356	0.018	4.750	0.735	1.929	21	-0.883
Vf_1-115	18.555	96.544	97.762	40.177	5.203	5.722	557	-59.495

Table 4 Statistical analysis of RQA results (t-Test)						
% DET						
	Yi vs Oi			Oi vs Vfi		
		P value	0.2245		P value	0.6504
		P value summary	ns		P value summary	ns
		Are means signif. different? (P < 0.05)	No		Are means signif. different? (P < 0.05)	No
		t, df	t=1.316 df=8		t, df	t=0.4708 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.0004		P value	0.0287
		P value summary	***		P value summary	*
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	Yes
		t, df	t=5.911 df=8		t, df	t=2.663 df=8
% Lam						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0021		P value	0.55
		P value summary	**		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=4.476 df=8		t, df	t=0.6241 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	P<0.0001		P value	0.0262
		P value summary	***		P value summary	*
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	Yes
		t, df	t=11.21 df=8		t, df	t=2.722 df=8
T.T.						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0201		P value	0.2161

		P value summary	*		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=2.894 df=8		t, df	t=1.343 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.0798		P value	0.7166
		P value summary	ns		P value summary	ns
		Are means signif. different? (P < 0.05)	No		Are means signif. different? (P < 0.05)	No
		t, df	t=2.006 df=8		t, df	t=0.3761 df=8
Ratio						
	Yi vs Oi			Oi vs Vfi		
		P value	0.4308		P value	0.0428
		P value summary	ns		P value summary	*
		Are means signif. different? (P < 0.05)	No		Are means signif. different? (P < 0.05)	Yes
		t, df	t=0.8296 df=8		t, df	t=2.406 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.647		P value	0.1523
		P value summary	ns		P value summary	ns
		Are means signif. different? (P < 0.05)	No		Are means signif. different? (P < 0.05)	No
		t, df	t=0.4757 df=8		t, df	t=1.582 df=8
Entropy						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0034		P value	0.5926
		P value summary	**		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=4.101 df=8		t, df	t=0.5572 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.0011		P value	0.0968
		P value summary	**		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=4.996 df=8		t, df	t=1.881 df=8
Max Line						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0013		P value	0.2956
		P value summary	**		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=4.859 df=8		t, df	t=1.119 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.0437		P value	0.4477
		P value summary	*		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No

		t, df	t=2.393 df=8		t, df	t=0.7984 df=8

Table. 5 Calculation of Variability of R-R signals by CZF method.

Subject	**(total variability-sec)**	**VLF (sec^2)**	**LF (sec^2)**	**HF (sec^2)**	**LF/HF**	**VLF/(LF+HF)**
	VT	**VLF**	**LF**	**HF**		
normal						
Y_1	1.398	0.113	0.306	0.619	0.495	0.123
Y_2	1.783	0.207	0.541	1.072	0.505	0.128
Y_3	1.228	0.087	0.263	0.448	0.588	0.122
Y_4	2.057	0.404	1.006	1.743	0.577	0.147
Y_5	1.239	0.103	0.291	0.520	0.559	0.127
O_1	0.756	0.040	0.102	0.189	0.541	0.136
O_2	0.640	0.009	0.031	0.099	0.314	0.072
O_3	0.711	0.029	0.085	0.179	0.473	0.108
O_5	0.577	0.022	0.058	0.121	0.479	0.120
O_6	0.817	0.044	0.127	0.248	0.512	0.116
Ventricular Tachycardia						
Vt_3-13	3.214	0.072	0.445	1.714	0.260	0.033
Vt_2-67	2.453	0.199	0.705	1.839	0.384	0.078
Vt_1-26	2.818	0.385	1.076	2.221	0.484	0.117
Vt_1-15	5.562	3.397	0.276	2.779	0.099	1.112
Vt_1-03	2.141	0.113	0.439	1.223	0.359	0.068
Ventricular Fibrillation						
Vf_2-30	3.106	0.451	1.272	2.608	0.488	0.116
Vf_2-71	2.833	0.361	0.993	2.110	0.471	0.116
Vf_1-8013	4.439	0.905	2.650	5.709	0.464	0.108
Vf_1-217	6.641	2.461	6.424	12.597	0.510	0.129
Vf_1-115	3.708	0.020	0.118	0.976	0.121	0.018

Table 6. Statistical analysis of results obtained by CZF method (t-Test)

VT						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0011		P value	0.001
		P value summary	**		P value summary	**

		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	Yes
		t, df	t=4.980 df=8		t, df	t=5.040 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.0032		P value	0.3497
		P value summary	**		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=4.162 df=8		t, df	t=0.9933 df=8
VLF						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0321		P value	0.0956
		P value summary	*		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=2.590 df=8		t, df	t=1.889 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.2464		P value	0.9936
		P value summary	ns		P value summary	ns
		Are means signif. different? (P < 0.05)	No		Are means signif. different? (P < 0.05)	No
		t, df	t=1.251 df=8		t, df	t=0.008276 df=8
LF						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0219		P value	0.0817
		P value summary	*		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=2.837 df=8		t, df	t=1.991 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.007		P value	0.1665
		P value summary	**		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=3.601 df=8		t, df	t=1.522 df=8
HF						
	Yi vs Oi			Oi vs Vfi		
		P value	0.0188		P value	0.0585
		P value summary	*		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=2.936 df=8		t, df	t=2.205 df=8
	Oi vs Vti			Vti vs Vfi		
		P value	0.0001		P value	0.2159
		P value summary	***		P value summary	ns
		Are means signif. different? (P < 0.05)	Yes		Are means signif. different? (P < 0.05)	No
		t, df	t=6.829 df=8		t, df	t=1.344 df=8

Table 7. Statistical analysis of results obtained by CZF method (correlation analysis)

	Correlation	Correlation	Correlation	Correlation	Correlation
	VT vs. VLF	VT vs. LF	VT vs. HF	VT vs. LF/HF	VT vs.VLF/(LF+HF)
Y_i	0.952 (*)	0.951 (*)	0.979 (**)	n.s.	n.s.
O_i	n.s.	0.879 (*)	0.923 (*)	n.s.	n.s.
Vt_i	0.953 (*)	n.s.	n.s.	n.s.	0.948 (*)
Vf_i	0.932 (*)	0.933 (*)	0.949 (*)	n.s.	n.s.

Table 8 Values of Hurst exponent

Subjects	H	D=2-H		Statistical analysis (t-Test)	
Y_1	0.070	1.930		**Y_i vs O_i**	
Y_2	0.125	1.875		P value	0.0142
Y_3	0.281	1.719		P value summary	*
Y_4	0.059	1.941		Are means signif. different? (P < 0.05)	Yes
Y_5	0.163	1.837		t, df	t=3.121 df=8
O_1	0.350	1.650		**O_i vs Vt_i**	
O_2	0.223	1.777		P value	0.0059
O_3	0.236	1.764		P value summary	**
O_5	0.319	1.681		Are means signif. different? (P < 0.05)	Yes
O_6	0.425	1.575		t, df	t=3.713 df=8
Vt_3-13	0.150	1.850		**O_i vs Vf_i**	
Vt_2-67	0.036	1.964		P value	0.0007
Vt_1-26	0.046	1.954		P value summary	***
Vt_1-15	0.240	1.760		Are means signif. different? (P < 0.05)	Yes
Vt_1-03	0.098	1.902		t, df	t=5.347 df=8
Vf_2-30	0.082	1.918		**Vt_i vs Vf_i**	
Vf_2-71	0.050	1.950		P value	0.4414
Vf_1-8013	0.021	1.979		P value summary	ns
Vf_1-217	0.152	1.848		Are means signif. different? (P < 0.05)	No
Vf_1-115	0.089	1.911		t, df	t=0.8099 df=8

Table 9 CZF: Analysis of brain waves from spontaneous EEG

delta <4 Hz	4<teta<8 Hz	8<alfa<12 Hz	12<beta<30 Hz	30<gamma<50 Hz	50-125 Hz
315830.18	1546.41	512.81	511.46	124.08	40.96
345604.54	1537.95	485.86	564.77	158.55	65.03
342601.77	1533.87	593.84	992.27	236.30	100.41
231064.75	1184.40	360.35	439.72	135.65	50.88
269108.24	1412.73	477.23	497.33	143.51	69.65
438748.26	2268.66	775.45	781.64	206.83	96.67
1157817.77	5349.84	1487.23	1438.89	395.36	181.99
770427.70	3858.46	1096.07	1095.69	335.10	127.57
296854.43	1635.37	561.97	592.92	177.03	107.97
420348.11	2272.28	769.46	799.45	243.02	135.93
462992.76	2266.45	694.40	773.47	264.10	105.22
855793.00	3727.05	1128.16	1258.53	474.90	209.62
625474.63	2916.07	871.99	882.63	234.88	108.78
430362.95	2232.10	735.97	829.91	261.08	123.09
979082.92	4177.67	1128.36	1435.87	481.32	175.21
882707.17	3041.39	986.53	1046.34	355.57	146.65

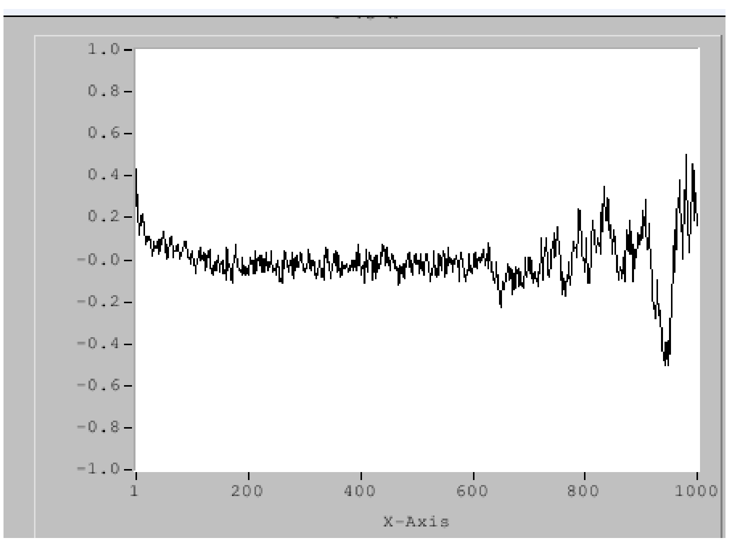


Figure 1a. Autocorrelation function of subject Y2.

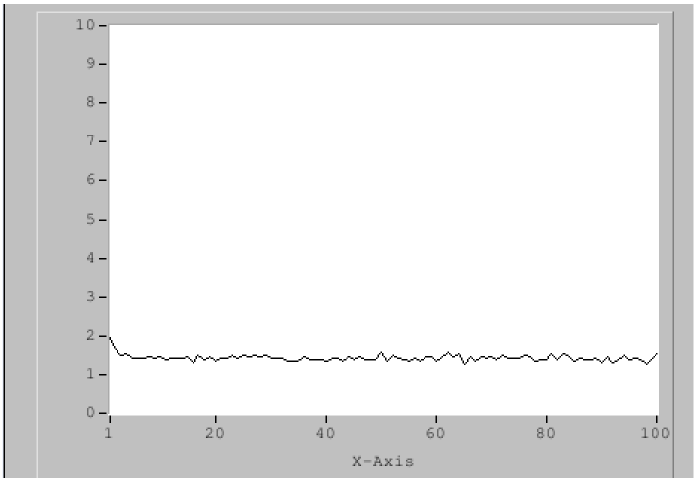


Figure 1b. Mutual Information of subject Y2.

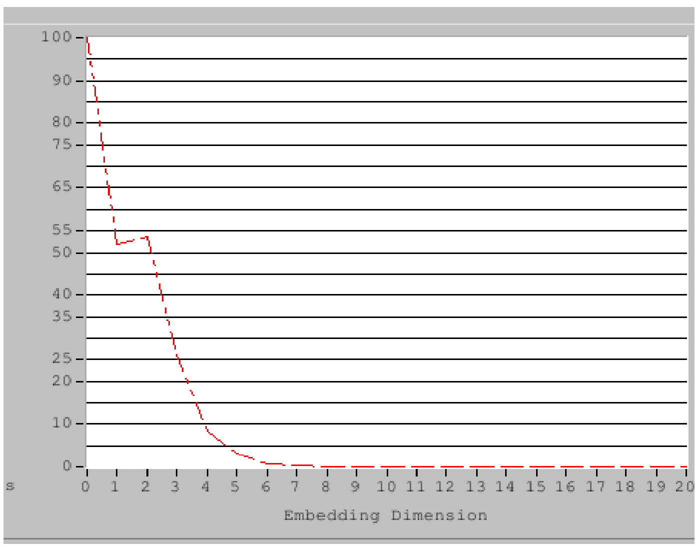


Figure 1c. False Nearest Neighbors of subject Y2.

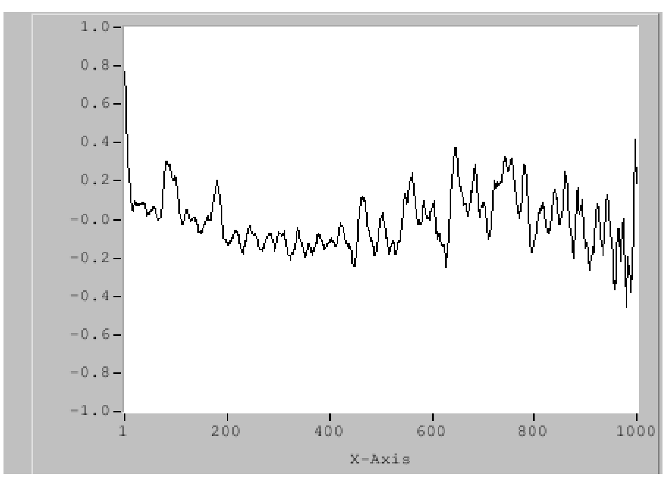


Figure 2a. Autocorrelation function of subject O3.

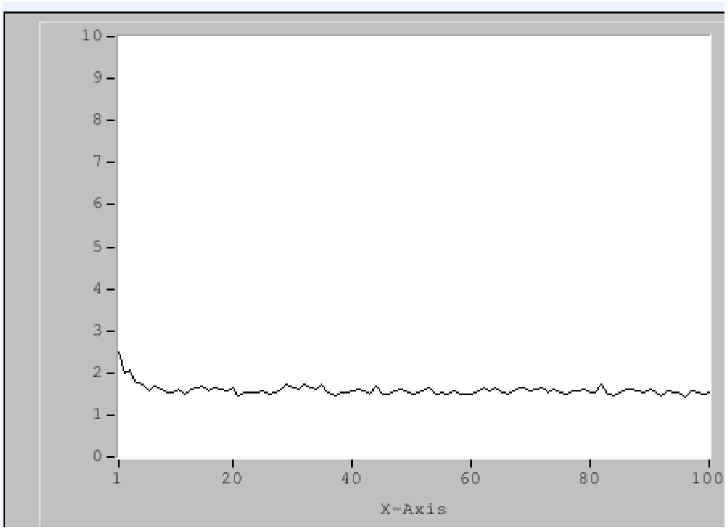


Figure 2b. Mutual Information of subject O3.

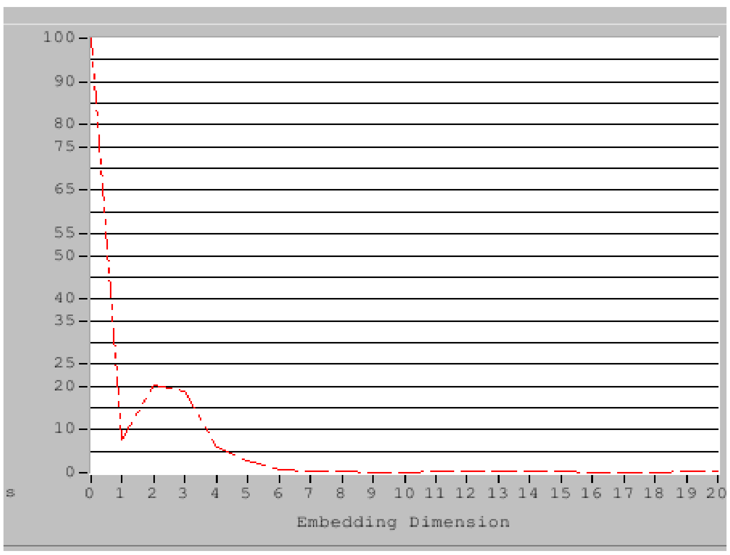


Figure 2c. False Nearest Neighbors of subject O3.

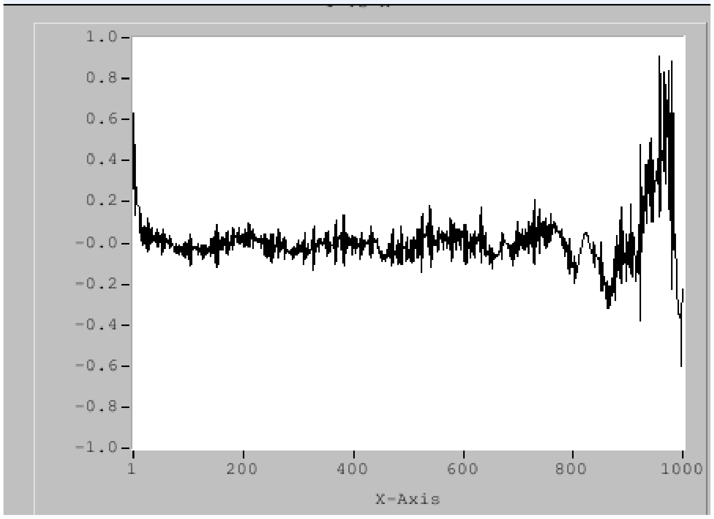


Figure 3a. Autocorrelation function of subject Vt1-26.

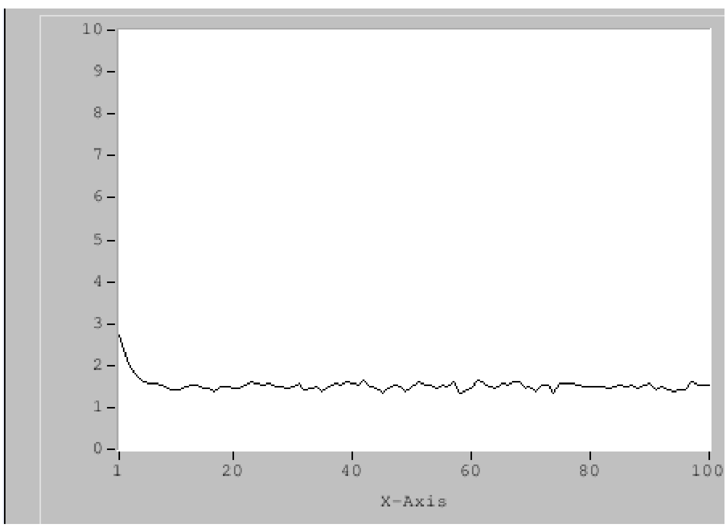


Figure 3b. Mutual Information of subject Vt1-26.

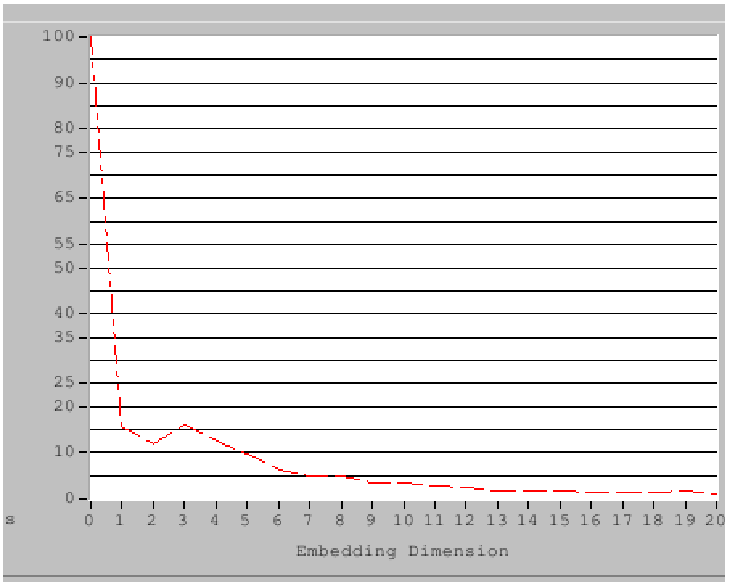


Figure 3c. False Nearest Neighbors of subject Vt1-26.

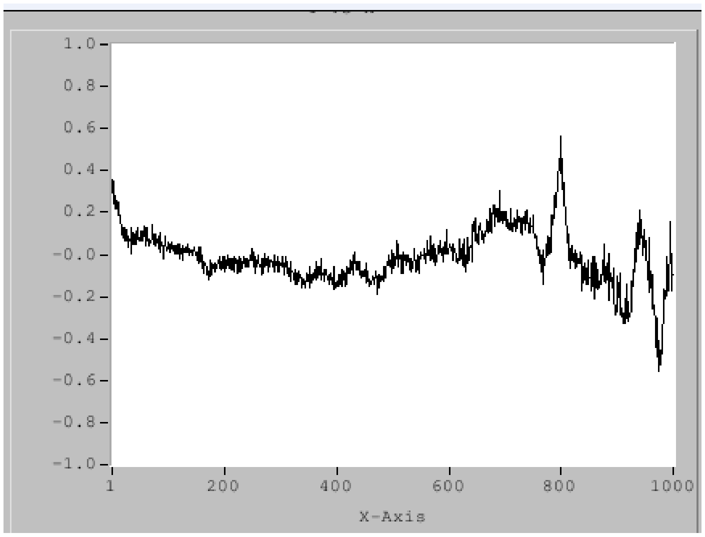


Figure 4a. Autocorrelation function of subject Vf-8013.

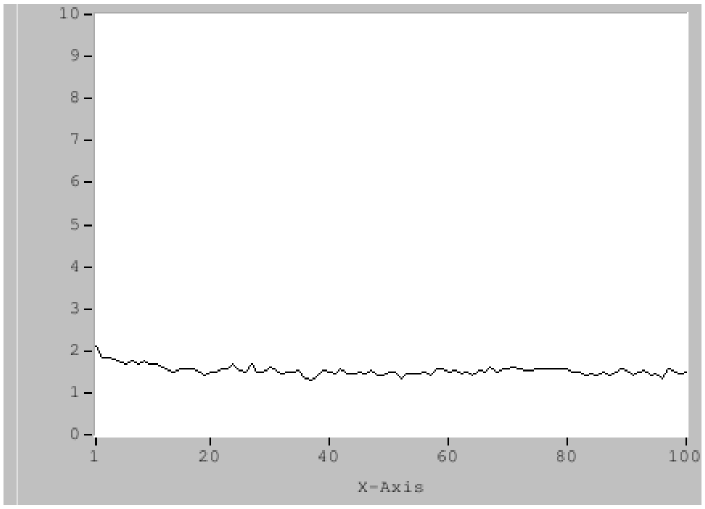


Figure 4b. Mutual Information of subject Vf-8013.

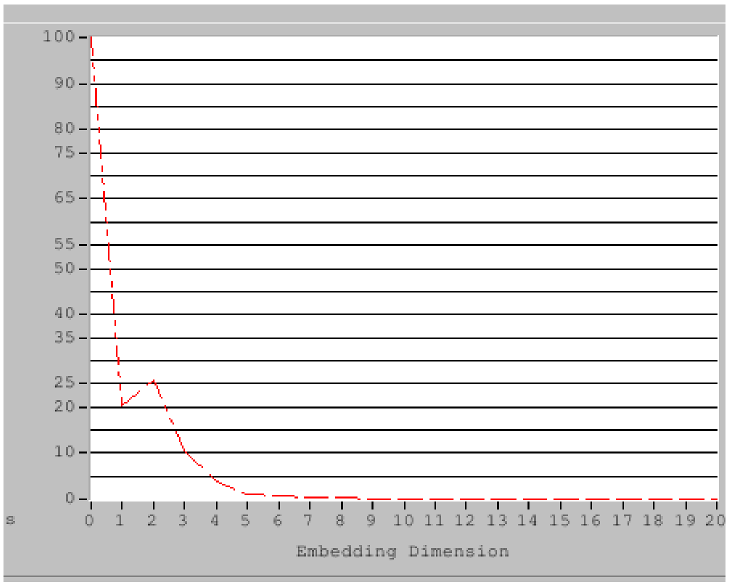


Figure 4c. False Nearest Neighbors of subject Vf-8013.

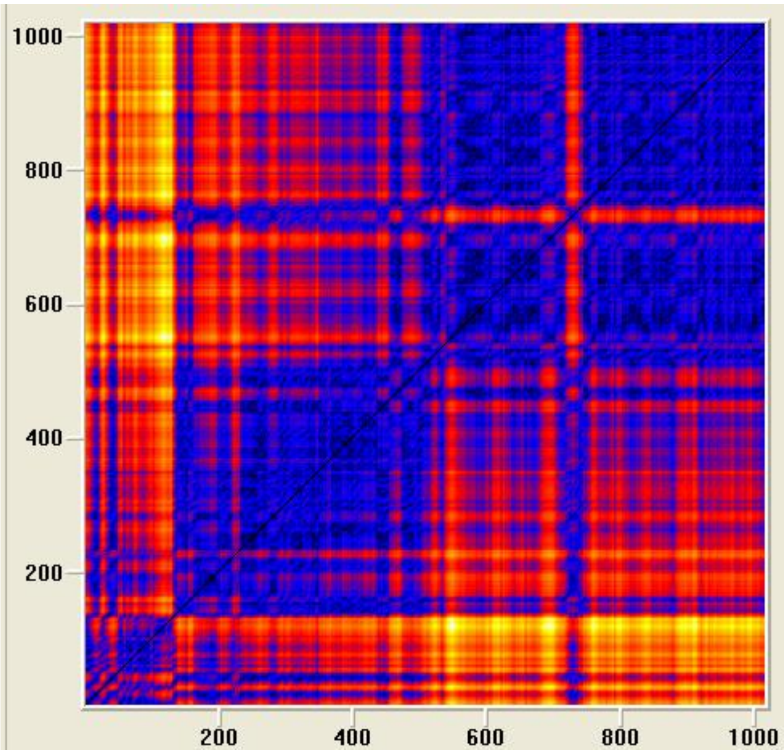


Figure 5. Recurrence Plot of the subject O_2.

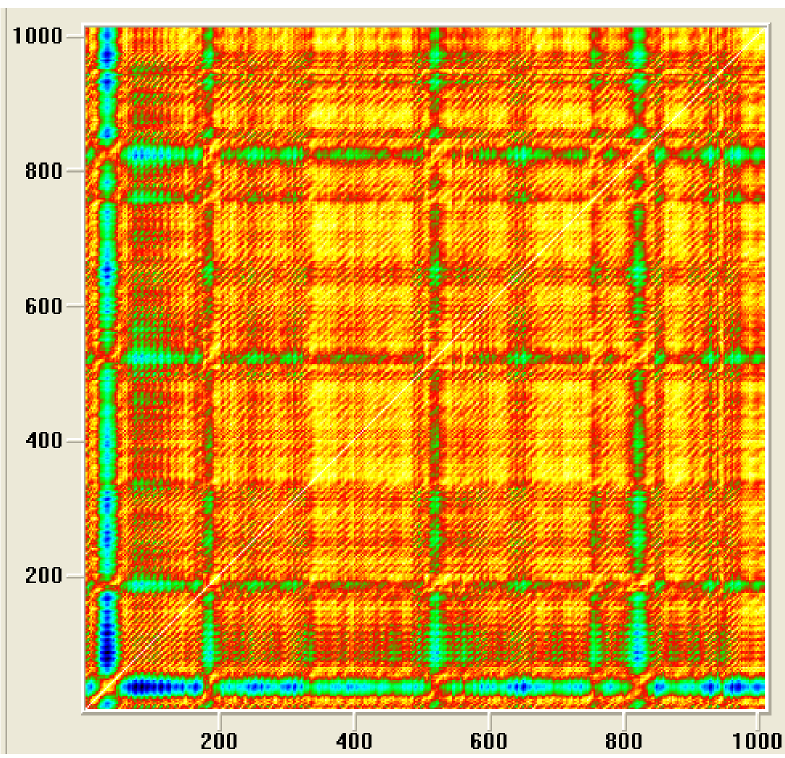


Figure 6. Recurrence Plot of the subject Y_3.

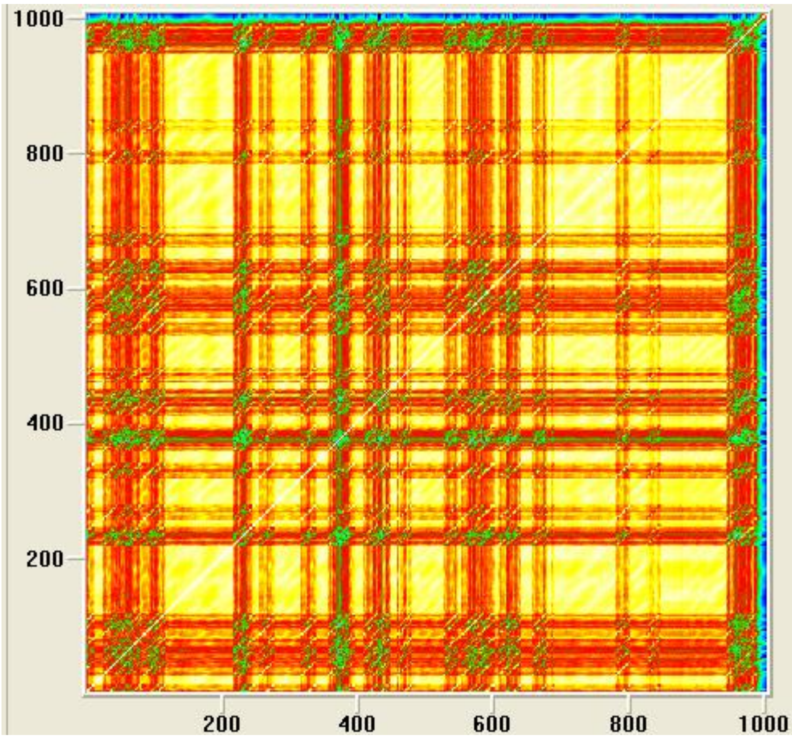


Figure 7. Recurrence Plot of the subject Vt_1-26.

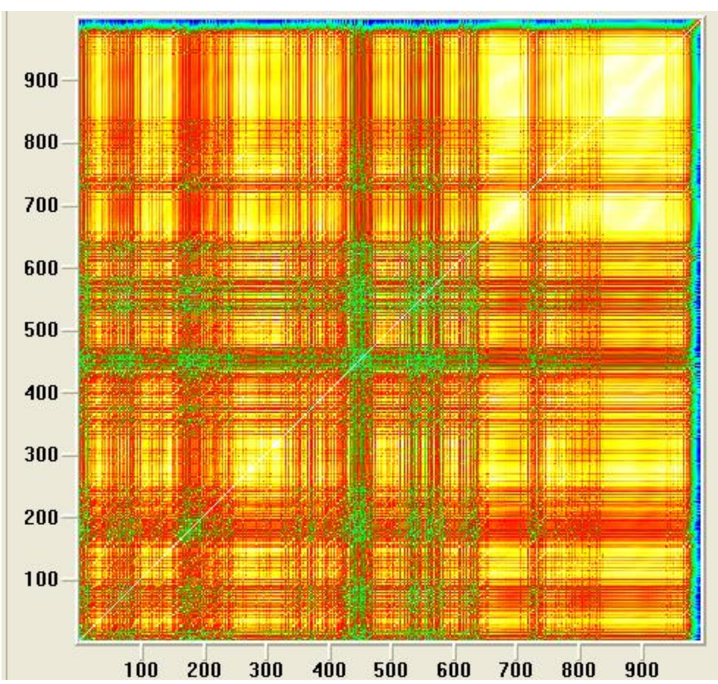


Figure 8. Recurrence Plot of the subject Vf_2-30.

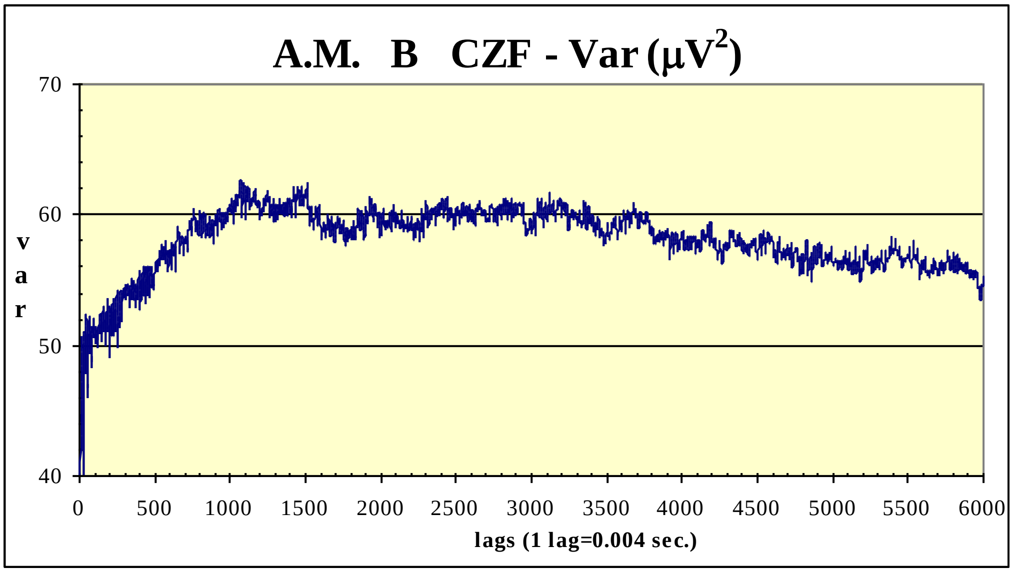


Figure 9. Variability analysis of spontaneous EEG in normal subject (A.M. B)

Figures for analysis of state anxiety

Fig. 1

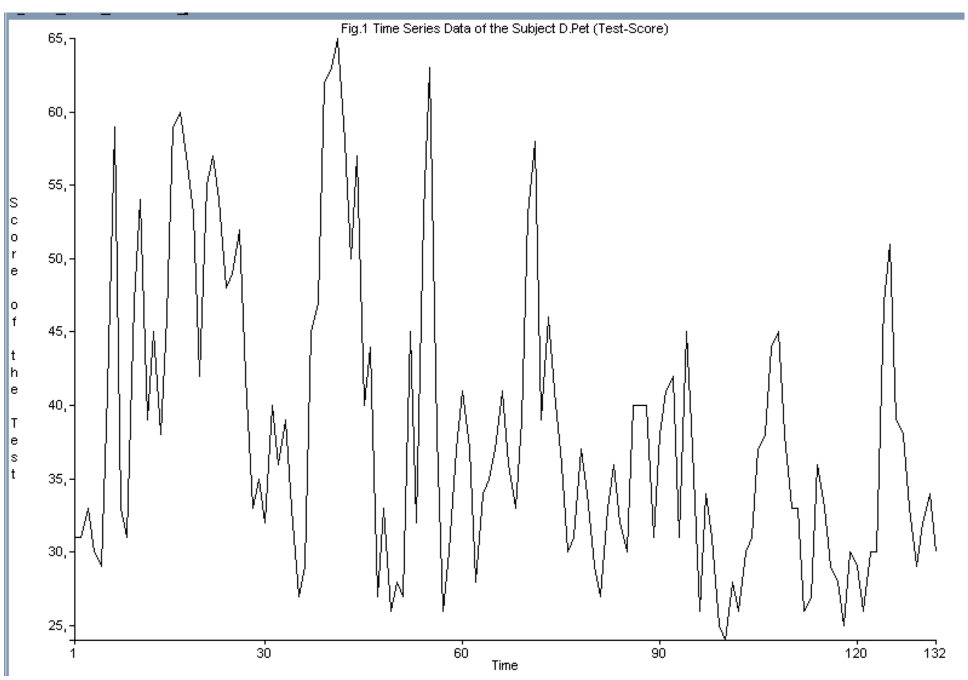


Fig.2: Subject F.Dav. - POINCARÉ PLOT
SD1 = 2.11
SD2 = 3.74

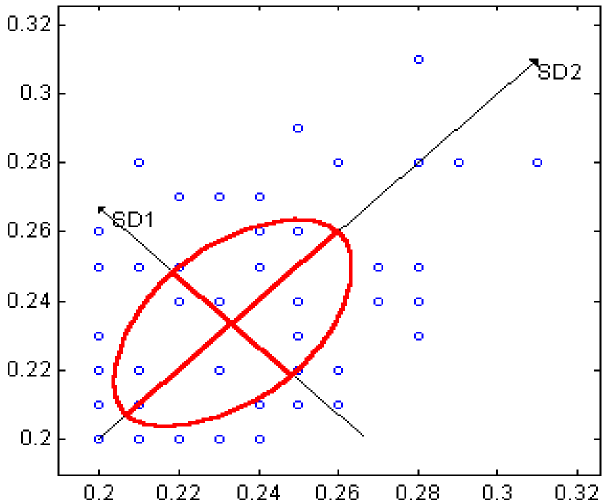


Fig.3: Subject A.Men. - POINCARÉ PLOT
SD1 = 2.21
SD2 = 3.13

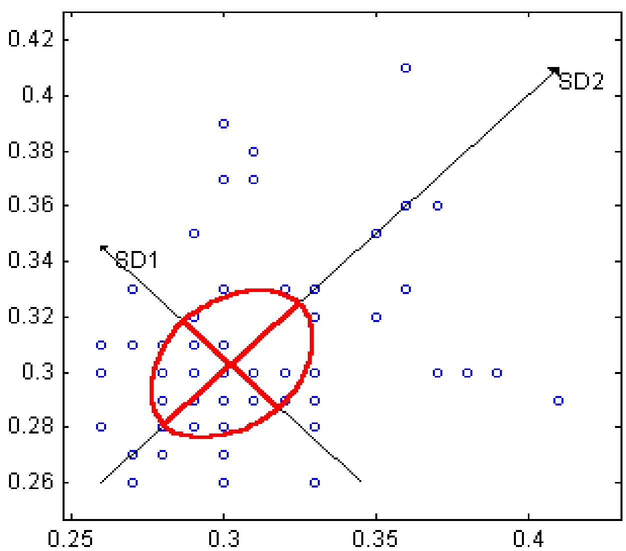


Fig.4: Subject A.Mac. - POINCARÉ PLOT
SD1 = 5.46
SD2 = 7.33

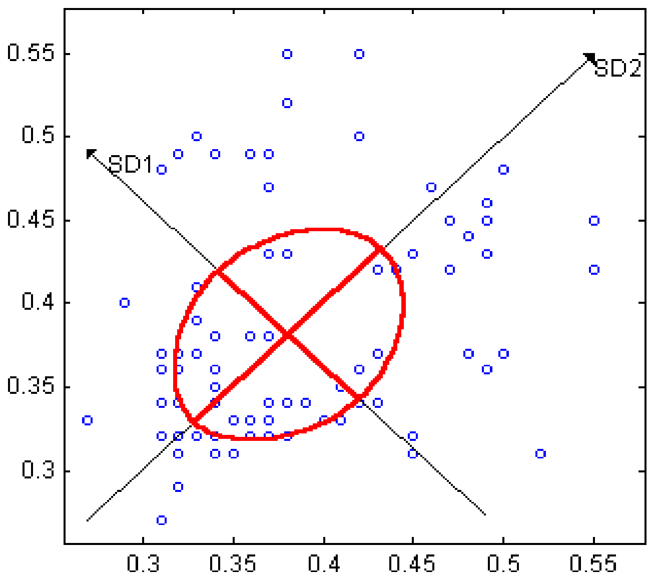


Fig.5: Subject D.Pet. - POINCARÉ PLOT
SD1 = 5.82
SD2 = 12.96

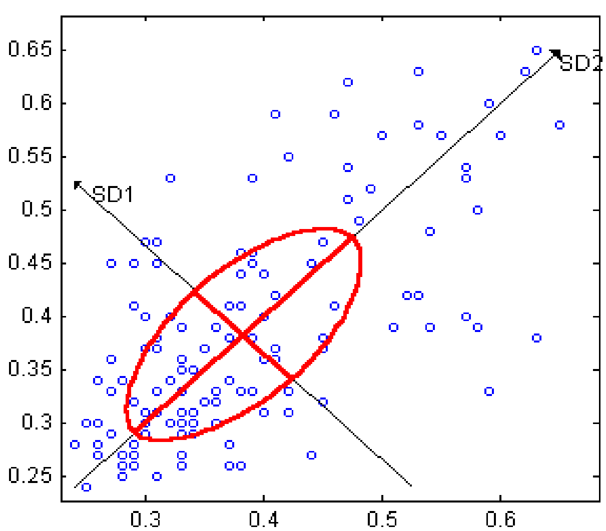


Fig.6: Subject M.Den. - POINCARÉ PLOT
SD1 = 9.12
SD2 = 12.64

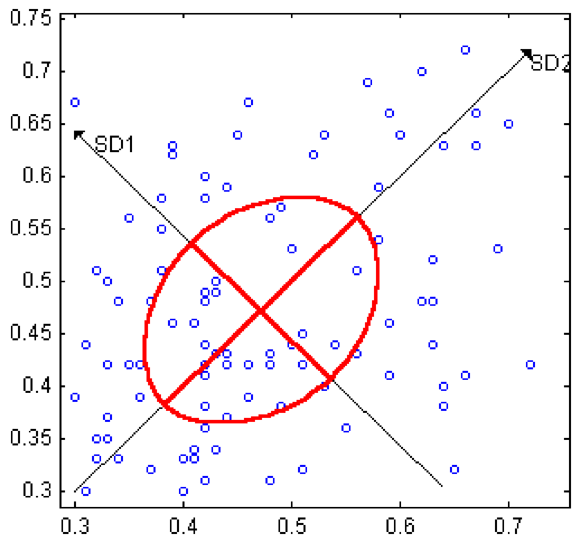
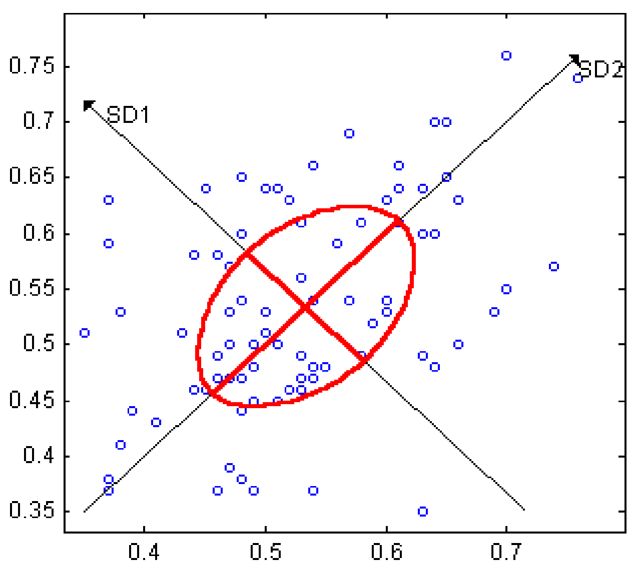


Fig.7: Subject G.Den. - POINCARÉ PLOT
SD1 = 6.89
SD2 = 10.96



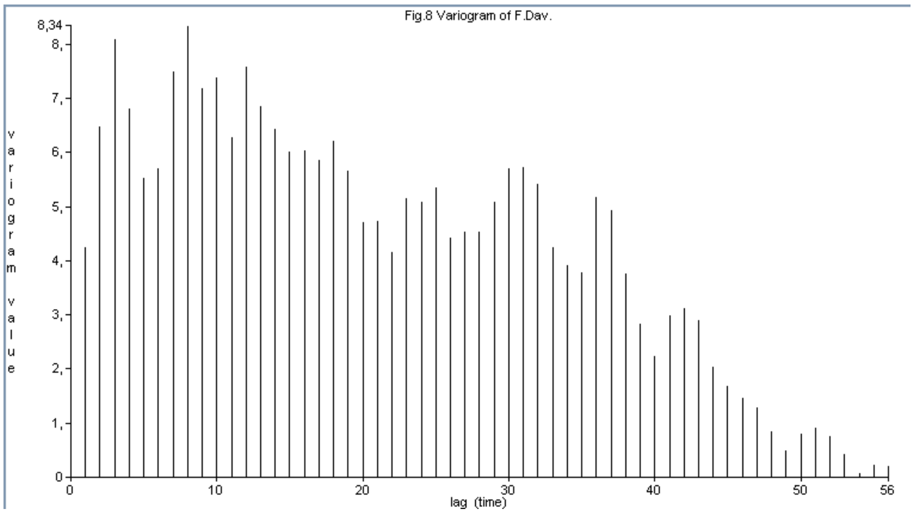


Fig.8 Variogram of F.Dav.
variogram value
lag (time)

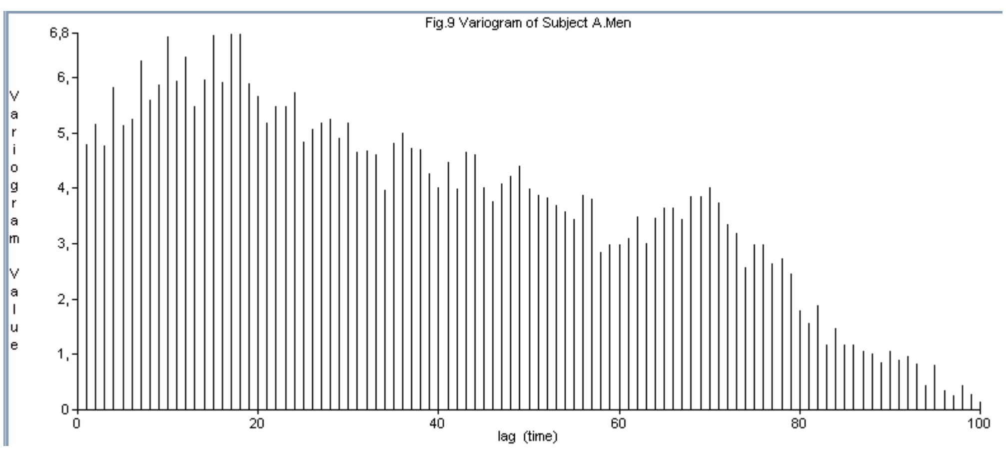


Fig.9 Variogram of Subject A.Men
Variogram Value
lag (time)

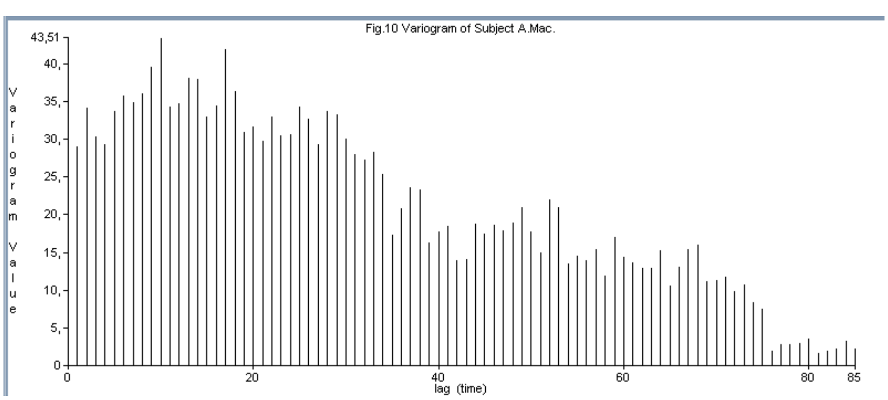


Fig.10 Variogram of Subject A.Mac.
Variogram Value
lag (time)

Fig.11 Variogram Analysis of Subject D.Pet

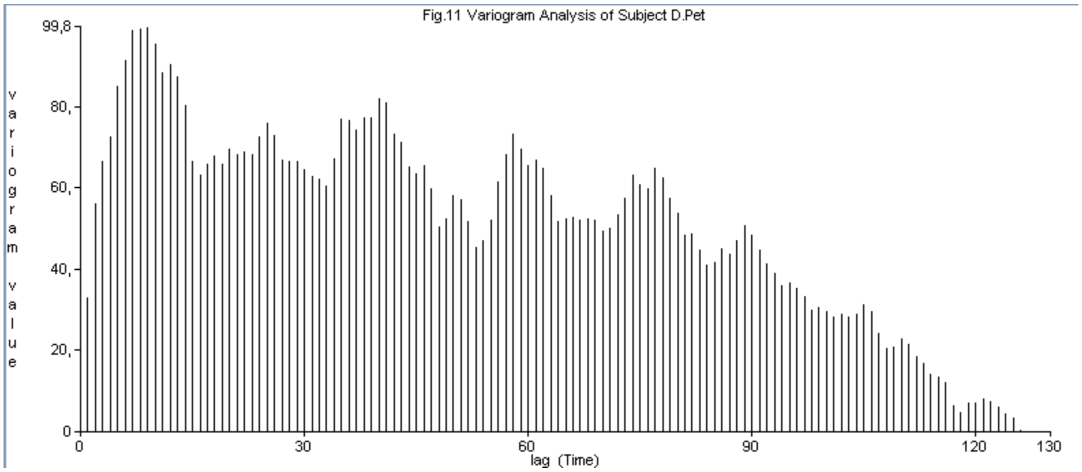


Fig.12 Variogram of Subject M.Den

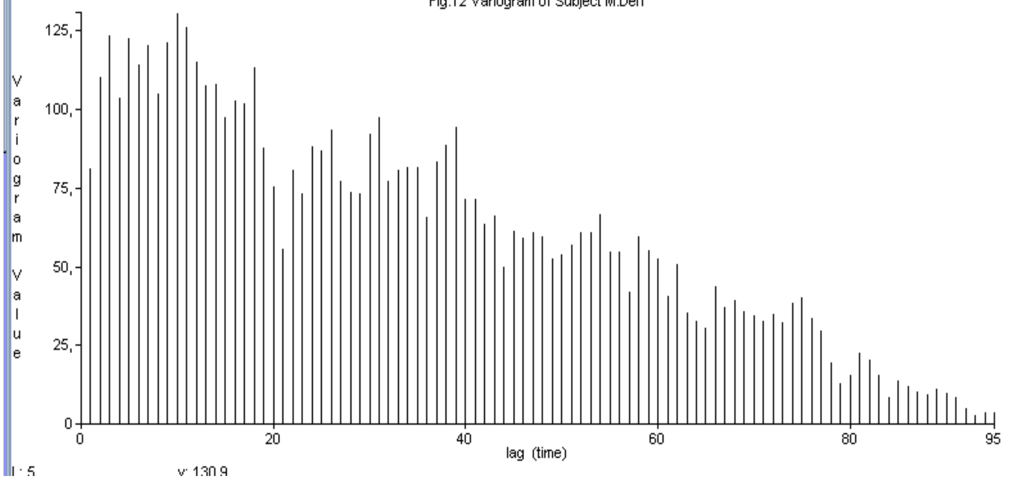


Fig.13 Variogram of Subject G.Den.

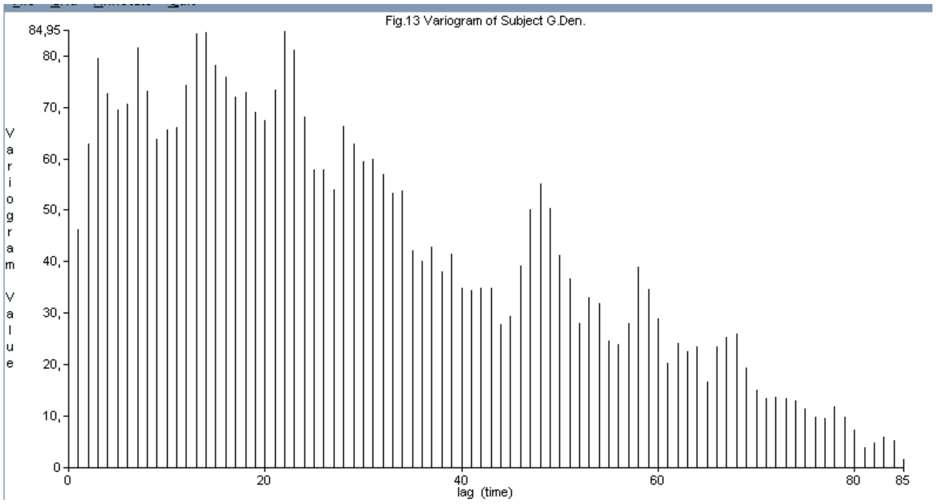


Fig. 14 Subject: F. Dav.

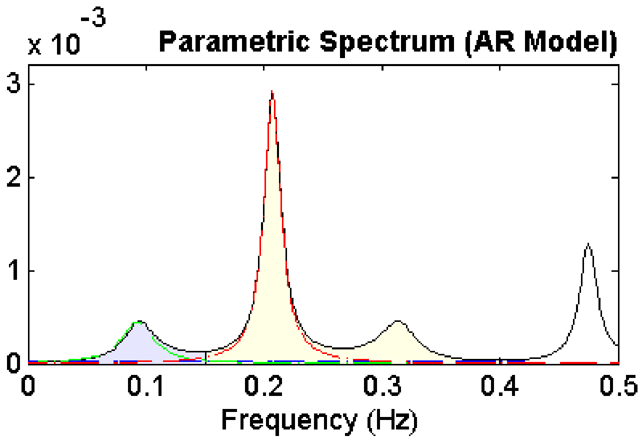


Fig. 15 Subject: A. Men.

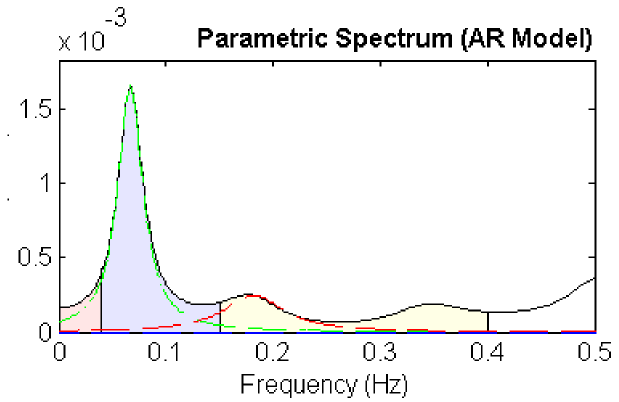


Fig. 16 Subject: A. Mac.

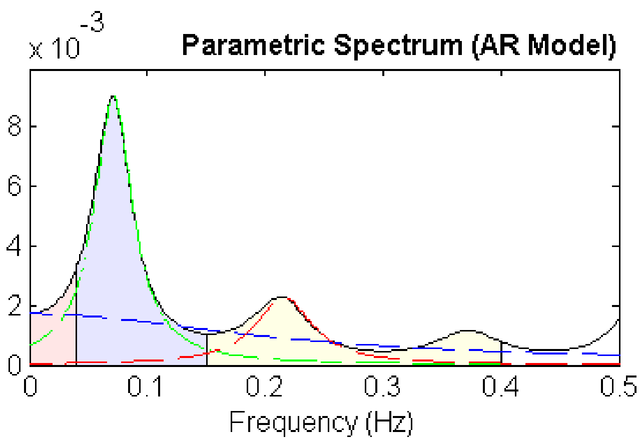


Fig. 17: Subject: D. Pet

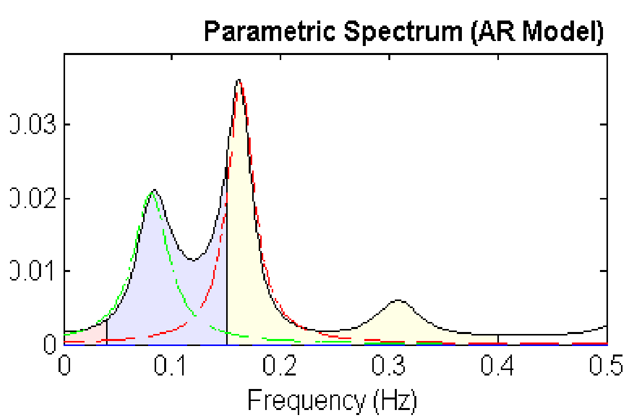


Fig.18 Subject M. Den

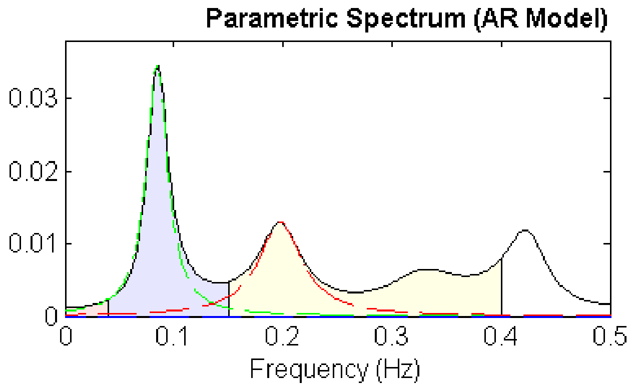
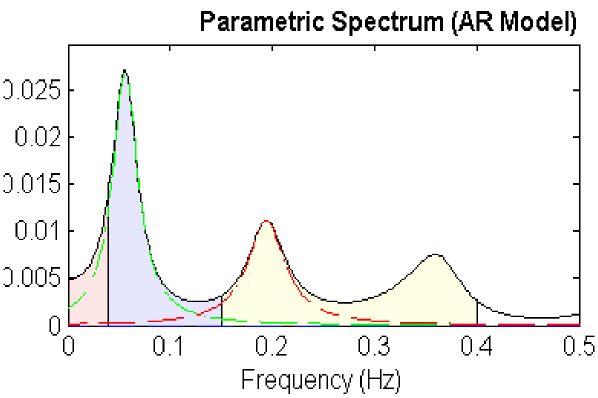


Fig.19: Subject G. Den.



Tab. 1 Statistics of Subject F. Dav

Mean	Median
23.62069	23.50000
Maximum	Minimum
31.00000	20.00000
StandardDeviation	Root Mean Squared
2.88174	23.79583
Variance	Skewness
8.30440	0.39860
Kurtosis	
-0.82782	

Tab. 3 Statistics of Subject G. Den

Mean	Median
53.39773	53.00000
Maximum	Minimum
76.00000	35.00000
StandardDeviation	Root Mean Squared
9.09238	54.16631
Variance	Skewness
82.67136	0.14723
Kurtosis	
-0.47710	

Tab. 2 Statistics of Subject D. Pet.

Mean	Median
38.31818	36.00000
Maximum	Minimum
65.00000	24.00000
StandardDeviation	Root Mean Squared
9.99494	39.60028
Variance	Skewness
99.89876	0.86033
Kurtosis	
-0.12236	

Tab. 4 Statistics of Subject A. Mac.

Mean	Median
38.10227	37.00000
Maximum	Minimum
55.00000	27.00000
StandardDeviation	Root Mean Squared
6.39254	38.63480
Variance	Skewness
40.86454	0.79387
Kurtosis	
-0.25014	

Tab. 5 Statistics of Subject A. Men.

Mean	Median
30.28431	30.00000
Maximum	Minimum
41.00000	26.00000
StandardDeviation	Root Mean Squared
2.71300	30.40559
Variance	Skewness
7.36034	1.61347
Kurtosis	
2.97502	

Tab. 6 Statistics of Subject M. Den.

Mean	Median
47.17526	44.00000
Maximum	Minimum
72.00000	30.00000
StandardDeviation	Root Mean Squared
10.94410	48.42807
Variance	Skewness
119.77341	0.45102
Kurtosis	
-0.80755	

Tab. 7: SD1 and SD2 Values calculated by Poincaré-Plots

Subject Name	SD1	SD2	Test-Mean Value	Standard Deviation	Variance
F. Dav	2,11	3,74	23,40	2,70	8,30
A. Men	2,21	3,13	30,30	2,40	7,36
A. Mac	5,46	7,33	38,10	6,10	40,86

D. Pet	5,82	12,96	38,30	9,00	99,89
M. Den	9,12	12,64	47,20	10,30	119,77
G. Den	6,89	10,96	53,40	8,80	82,67

Tab. 8: Fractal Analysis

Subject Name	Fractal Measure	Generalized Fractal Dimension
F. Dav.	13,200	-0,350
A. Men.	17,500	-0,397
A. Mac.	108,400	-0,450
G. Den.	230,310	-0,495
D. Pet.	269,300	-0,420
M. Den.	305,000	-0,420

Tab. 9 Frequency Domain Analysis

Subject Name	Frequency range (0.1Hz) - Power Spectrum (Test Score)	Frequency range (0.2Hz) - Power Spectrum (Test Score)	Frequency range (0.3-0.4Hz) - Power Spectrum (Test Score)	Frequency range (0.5Hz) - Power Spectrum (Test Score)
F. Dav.	0.00050	0.00300	0.00050	0.00150
A Men.	0.00150	0.00025	0.00018	0.00050
A. Mac.	0.00900	0.00200	0.00180	0.00200
D. Pet.	0.02000	0.03500	0.00500	0.00250
M. Den.	0.03500	0.01500	0.00500	0.01000
G. Den.	0.02500	0.01000	0.00500	0.00010