

The Infinite Geometry of The Mind

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“Sitting Under the Plum Blossom How Strange.”

Sengai – (Haiku)

The (brief) experience of awe that is caused by a consciousness that catches a brief glimpse of itself and refuses to be ‘put in its place’ is unmediated and revelatory.

- I will try and relate the geometry of the (empty) mind to infinity, by briefly exploring sacred spaces and cultural connections between consciousness, awe and infinity.
- I will argue that both consciousness and infinity are ‘awe’ inspiring because they cannot be contained.
- I will argue that questions about the constitution of consciousness can benefit from questions about the constitution of space.
- I will argue that an Anti de Sitter space structure that is ‘finite from without but infinite from within’ is well suited for the egocentric space of the mind, allowing for the possibility that we are ‘island universes’ of sorts.

I was recently in a talk by Takuya Niikawa and Uriah Kriegel about the phenomenology of the sublime and the awe-inspiring nature of consciousness, where among other things, he related the awe generated by the encounter with consciousness to the awe generated by the encounter with infinity, which he tried to create with a beautiful slide of distant galaxies.² Unfortunately, we lack a well worked out ‘phenomenology of awe and wonder’ or one of the related phenomenology of ‘horror’. Some questions that need to be answered:

- What is the phenomenology of awe? (mood, propositional attitude, visceral).
- What is the phenomenology of infinity? Or wonder? Or the wonder of infinity?
- Does awe conflate wonder and horror?
- How do we experience the awe that is sometimes caused by consciousness?
- What is the ontology (actual?) and epistemology of infinity?

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² The talk was at the *Kobe Philosophy of Consciousness Conference*, December 13, 2023, Kobe University, Kobe, Japan. Also see recent publication by Niikawa & Kriegel (2025).

We also harbor a powerful intuition that the conscious mental state is not bounded, reminiscent of the Archimedean intuition – if $x > y$ there always exists a number n such that $x < ny$.

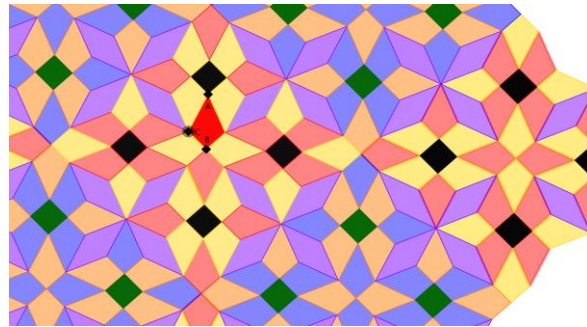


Figure 1. Shows an example of the Tessellation of the plane.
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In “To Infinity and Beyond: A Cultural History of The Infinite” – a book by Eli Maor (Maor, 1991), we learn of the ‘Tessellation of the plane’, covering the infinite plane with a repeating pattern. It has been long established that there are exactly 19 kinds of ways of tiling the whole infinite plane with repeating tiles without leaving any ‘holes’ (Figure 1, shows an example).

This aversion to leaving any ‘unfilled’ spaces in a design (filling spaces with information or objects) is called “Horror Vacui”, derived from Latin, often translated as “the fear or horror of the empty, empty space or emptiness”. (see Figure 2 for another example).



Figure 2. ‘The Fall of Babylon’, by Jean Duvet. Apocalypse series, circa 1555 is an example of a ‘Horror Vacui’. Photo by unknown author, public domain.

Maor surmises that this expression of tessellation is related to the powerful intuition that reality cannot contain ‘holes’ (only as holes). It relates the ‘no boundary’ intuition to questions about the Continuum and to questions about Infinity. Maor considers projective geometry and the ‘one to one’ mapping between the unit circle and infinite line!

SACRED SPACES

Maor relates the tessellation of the plane to human cultural and religious attempts to present and apprehend infinity. Here we see that sacred spaces are awe(full/filled) spaces and that awful(l) spaces are infinite spaces. Sacred spaces capture the ‘infinite-in-here’.

Maor reminds us that the Blue and Gold that dominate mosques are the colors of infinity (see Figure 3). The minaret breaks from the mosque pointing out as a constant reminder of the beyond. The mosque is covered with tiles on the inside as an expression of infinity and Maor reminds us that the Moslems discovered 17 of the 19 ways of tessellating the plane!



Figure 3. Examples of Blue and Gold, ‘colors of infinity’, dominant in mosques mosaics. Photos by Unknown Author, licensed under [CC BY](https://creativecommons.org/licenses/by/4.0/).

A sacred space is designed to capture infinity and what we are interested in here is the possibility that these religious and cultural sacred spaces are desperately trying to remind us that we too are a sacred space. Here I think that the most relevant analogy for our purpose, that is to both consciousness and infinity, is that of ‘containing that which cannot be contained’, or ‘containing the uncontainable’. Uncontainable even by words; the sublime, the infinite and the ‘God’ that resists description.

The sacred spaces that fill us with awe are a metaphor of our inner space as uncontainable by ordinary space.

Synagogues too are sacred spaces. When entering one, inscribed above one’s head, in Hebrew, is the sentence “Ma Nora Hamakom Hazeh”, “How ‘Nora’ is this place”. Nora is an ancient word (Pre-Ugaritic, possibly Pre-Amorite root) that conflates Horror, Awe and Wonder with the ‘limit of limiting’, to remind you that you are entering an unbounded sacred space.

Yehuda Amichai, a prominent Israeli poet, was asked about the purpose of the blue stripes on the Jewish Tallit, and he said that stripes, unlike squares, represent infinity and that the blue stripes serve as landing strips for the angels. I heard this quoted by a Rabbi at a friend’s (Aharonov) grandson’s Bar Mitzvah. There is a more esoteric connection here between the Lurianic Kabbalah’s view of both Man and the World as infinite (on the inside) containers of light, but that would take us too far afield. There are also references by The Zohar to a ‘circular yet infinite ray of light’ (see Baal Hasulam, Rabbi Yehuda Ashlag) that goes well with the aforementioned *one to one* projection of the unit circle on the infinite line, reminiscent of the mystical religious traditions

and Rabbi Itzhak Luria's Kabbalistic fragment – “Man is little World and World is big Man”. However, ‘world’ in Hebrew is ‘OLAM’ that is related to both NEELAM, which means ‘disappears’ and TAALUMA, which means mystery and is more similar to the pre-Socratic take on the world and the Heraclitean fragment - ‘Nature likes to hide’.

Interestingly, we have recently discovered that there are more ways to tessellate the plane than the 19 ways previously established. In 1982, Dan Schechtman, an Israeli physicist, discovered a crystal structure with pentagonal symmetry that was considered impossible by the theorists (Clery, 2011). He was ostracized and almost kicked out of his university for this scientific heresy, but he refused to withdraw his results and eventually won a Nobel prize for discovering the so called ‘quasi-crystals’, after Roger Penrose established their theoretical possibility (Austin, 2005). See Figure 4 for an image of Penrose Tiling.

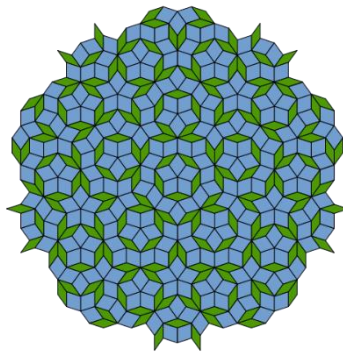


Figure 4. A [Penrose tiling](#) with rhombi exhibiting fivefold symmetry. Image from unknown source, in public domain, licenced under CC BY-SA.

The amazing end of this story is, that sometime after the discovery of this exotic never repeating way of tessellating the plane, it was found that the Muslims in The Darb-i Imam shrine (1493 AD) had already discovered the ‘Penrose Tiles’ (Wilford, 2007) (see Figure 5).



Figure 5. The Darb-i Imam shrine – 1493 AD, Muslims first to discover the Penrose tiles.

ON THE (DIS)UNITY OF CONSCIOUSNESS

Gilbert Ryle famously said (albeit sarcastically) – “*Absolute solitude is on this showing the ineluctable destiny of the soul. Only our bodies can meet.*” (Ryle, 2009), p. 5, Chapter 1. However, the abyss separating conscious beings is crucial to our subjectivity and sense of sovereignty, and also crucial to the ‘other mind’s problem’. (Why does it seem that ascribing consciousness to the other demands a miracle?).

We can call the inability to access the other’s consciousness - ‘the disunity of consciousness’ - and agree that both the unity of consciousness and its disunity are two sides of the same coin, so to speak. It is not likely that one can understand the nature of the unity of consciousness without understanding the nature of its spatial demarcation condition, especially if, like me, one embraces substance physicalism and consciousness as an exotic phase of matter.

The yearning to overcome the abyss separating conscious humans, both from each other and from ‘God’, has occupied Sufi poets and religious mystics for generations.

Perhaps these sacred spaces that serve as ‘containers of the uncontainable’ were constructed by us to remind ourselves that we too harbor infinity, and that the experience of the ‘awe of the infinite’ that they elicit acts to remind us that we are ‘infinite beings’ in some sense.

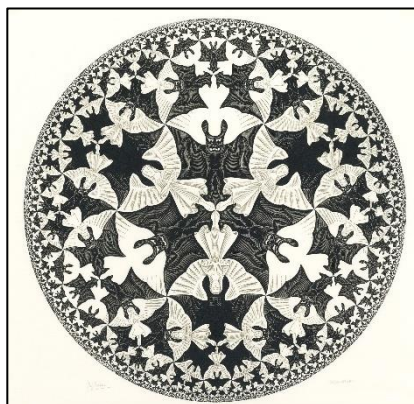


Figure 6. “*Circle Limit IV: Heaven and Hell*” by M.C. Escher (dated 1960). Photo by Unknown Author, licensed under [CC BY-NC-ND](#). Credit: Wiki Art.

‘Infinity’ can either be seen as a useful idealization constructed by humans or else as actual. I am more interested in the possibility that we harbor actual infinity, and this outrageous possibility can be broken into two slightly less outrageous possibilities. It’s possible that consciousness is constituted similarly to space and the continuum, and it’s possible that the continuum harbors actual infinity. I am not an expert on the ontology or phenomenology of the infinite, but rather ask a more ‘physicalist’ question, ‘Is it possible that consciousness is a strange phase of matter that is constituted like space?’ And also ‘In what sense would that ‘space’ be infinite?’ If consciousness is constituted like space, does this make us ‘island universes’ of sorts? The question I am curious

about then, is whether we are ‘infinite’ in the sense that a universe, or a patch of the continuum, might be infinite, whether actual or not.

I believe that recent breakthroughs in modern physics that shed light on the constitution of space, especially ones related to the Holographic Correspondence (Maldacena, 1999)³ and the recently discovered connections between the local robustness of space and nonlocal quantum error-correction codes, make it possible to conceive of ways - in which the brain’s neural correlates of consciousness (NCC) generate something akin to the continuum – a bit easier.

The holographic correspondence or the Anti De Sitter/Conformal Field Theory known as the (AdS/CFT) duality, provides two ‘shockingly’ (Castellani & Rickles, 2017) different ways of describing the same system, one, a d-dimensional quantum field theory (QFT) made up of entangled Qbits, and the other, a d+1 – dimensional classical, continuous Anti de Sitter (AdS) space with gravitation.

This means that if, were the brain massively entangled (for some strange reason), say in a two-dimensional membrane, like the Claustrum (possible NCC) (Calarco, 2024), and describable by a conformal QFT (or CFT), it would ‘generate’ a three dimensional classical, negatively curved, continuous AdS space. Or what Juan Maldacena calls ‘infinity in a bottle’.

The structure of the geometric AdS space is much more similar to the structure of our phenomenological field than the structure of the CFT, which is made up of a gigantic Hilbert space with a huge number of entangled Qbits and can be so opaque that the only way to make sense of it is by switching to the simpler dual AdS space. Physicists were actually forced to do that to solve intractable problems in condensed matter theory leading to the discovery of ‘strange metals’ (Hartnoll, Kovtun, Muller, & Sachdev, 2007).

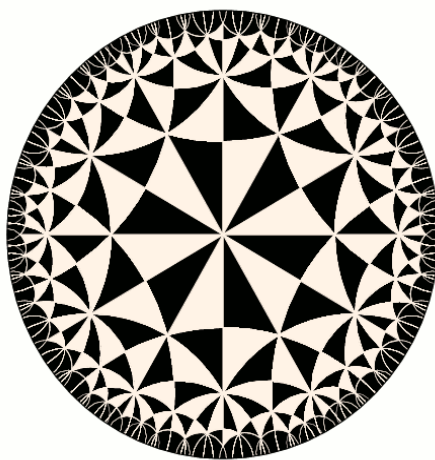


Figure 7. An example of Two Di AdS space. “Crystal Symmetry and its Generalizations”, from H.S.M. Coxeter’s address to the Royal Society of Canada (vol. 51, 1957) with Escher’s pencil studies.

³ The duality was formulated in 1997.

In a thought experiment by Leonard Susskind, a large ‘spherical shell’ quantum computer (in a lab) ‘generates’ a three-dimensional AdS space that is dual to the shell’s CFT. That AdS space is different than the lab space inside the shell (2-Di)! For Susskind the correspondence is a metaphysical necessity, for others like Verlinde, the correspondence is more similar to Thermodynamic reduction, and yet for others a case of theoretic overdetermination. Tiziana Visterini (Castellani & Rickles, 2017) even likens the AdS/CFT to a dual aspect theory of information, we just need it to be strange enough ontologically, perhaps loosening up the ties between identity and necessity (Levine, 2019). According to Susskind, the only way to access that AdS space is by uploading a ‘technician’ onto the shell computer. The ‘technician’ need not be conscious however, as we will see, if it were conscious and that consciousness was constituted like space, then, since the local robustness (stability) of that space is determined by a Nonlocal Quantum Error-Correction Code (NQECC), that code cannot be made to merge with the quantum computer’s NQECC.

Imagine a large book with many pages, each with many fluctuating words, with a bad signal to noise ratio. However, the meaning of each word is determined by the meaning of many other words on other pages with which it is entangled, i.e., it is nonlocal. When information content is degraded locally because of fluctuations, it can be restored because its meaning is nonlocal and scattered all over the book. If the local robustness of space is really guaranteed by the action of such NQECCs, then such spaces cannot be simply joined or cut into smaller pieces. Cut a space in two and you lose so much nonlocal information that it impairs the local robustness of the space. Try and join two spaces with different NQECCs and those cannot be made to merge. It is not as if you take two vacuum volumes and just join them. Also, it is quite impossible to create a combined space with a single shared NQECC without destroying the original spaces. From another perspective using General Relativity, the topologist Robert Geroch proved in 1978 that cutting space in two has temporal consequences resulting either in temporally non-orientable manifolds or closed time-like curves. See the fascinating history that led to Geroch’s discovery (both Descartes and Kant thought that space could not be cut in two (Callender & Weingard, 2000). Geroch’s proof suggests that were consciousness constituted like space its spatial demarcation conditions would come with a unique internal temporal architecture.

What is interesting here is that if consciousness is similar to space as suggested (assuming that the brain is massively entangled for biologically relevant duration), one gets rid of many philosophical problems that are related to the mind-body problem. Including:

- a) Impossible identity paradoxes. (Scott Aaronson).
- b) Subject addition combination problem.
- c) The other minds problem.
- d) Privacy and the subject.
- e) No uploading of conscious agents.
- f) The spatial demarcation conditions of consciousness.
- g) Its relationship to real space (overcoming David Lewis’ rejection of “Disconnected spacetimes”) (Lewis, 1986).
- h) Defends physicalism from Conceivability Argument. (Awret, 2024)
- i) You get geometry for free.
- j) Constitution of consciousness.

k) Solution of structural mismatch problem.

So what I claim is that a very strange empirical scenario has so many significant philosophical benefits (I can count more than ten!) that it is worth exploring. One unintended consequence of this approach is that it suggests a way in which we can be finite from without and infinite from within.

CONSCIOUSNESS AS SPACE - 7 ARGUMENTS

Motivating the argument that consciousness and space are similar enough to learn about the constitution of consciousness from the constitution of space needs some work and I will provide a quick sketch of seven additional arguments.

1. Argument from philosophy:

- Formal similarities between the hard problem of space (Le Bihan) and the hard problem of consciousness.
- Realism (substantivism) vs. Eliminativism. You can be a realist or an eliminativist on both space and consciousness.
- Functionalism on consciousness and Functionalism on space as in Causal Set theories of space.
- Both problems face space related measurement problems, or an ‘Empirical coherence problem’ and an irreducible need to borrow from that which they need to explain.
- Showing that space and time emerge from something that is neither spatial nor temporal weakens theories of mind that are based on the rejection of strong emergence (Galen Strawson).

2. Argument from novel physics:

- Consciousness is a unique phenomenon.
- The brain is a unique physical system.
- The two are exquisitely correlated.
- The brain is ideally suited to instantiate novel and strange physics.
- The putative physics of consciousness must be very strange.
- Conclusion: We are missing strange relevant physical brain facts.
- Such radical facts are usually discovered during scientific revolutions.
- AdS/CFT is part of a scientific revolution.

3. Argument from background independent physics:

- Physical existence is spatiotemporal existence (Non-spatiotemporal existence is reminiscent of dualism).
- We have no idea how to relate the subjective conscious space (the elusive ‘space of consciousness’) of appearances to our ordinary laboratory space.
- This results in a powerful anti-physicalist intuition.
- One way out is to look for a physics whose constituents do not exist in space but that exist nevertheless!
- A natural candidate – The physics of space itself, in which space emerges from more fundamental existing constituents (networks of entangled Qbits do not float ‘in space’ like ideal gas molecules).

4. Argument from Complexity:

- Anderson - More is different seems to be true for the generation of novelty in both CMT (Condensed Matter Theory) and computation, as we see from large language model (LLMs).
- The brain is an incredibly complex, hierarchically nested physical system exhibiting strange properties.
- The new and surprising connections between complexity, computability, information and quantum mechanics, especially CMT and many body theory which currently provide our most advanced quantum field theory.
- What John Preskil terms the ‘New Frontier’ navigating gigantic Hilbert Spaces. Where one gram of entangled matter inhabits a space with $(10 \exp(10 \exp 23))!$ states.
- The unexplored new field of quantum complexity may provide us with new laws similar to the purely theoretical Bekenstein Bound on black hole formation.
- It may mean that as a system’s complexity exceeds a certain bound, it becomes indistinguishable from other such ‘complex enough’ systems.
- If the principle of ‘loss of which way’ information essential to coherence in quantum optics could also result from exceeding a complexity bound, then the indistinguishable causal histories in the NCC’s operation could become massively entangled, which is crucial for the CFT side of the equation.

5. Argument from medium:

- Physical realization of information demands a robust, enduring and neutral medium.
- We don’t have a physical theory of realization.
- Consider the next three fundamental mediums:
 - a) Elementary particles are realized in space. (Excitations of fields in QFT).

- b) Quasi-particles are realized in the ground state of many particle systems in CMT.
- c) Conscious representational content is realized in some brain medium, or stage, in which information is realized physically (I assume).

Fundamental mediums are rare, do these three have something in common? Yoshiro Nambu proved that the QFT vacuum and the CMT ground state are mathematically identical.

We relate the vacuum in QFT to the many particle ground state of CMT and ask whether the relevant CMT is instantiated by the brain. If it is, we move from the brain CMT to its AdS dual with all the benefits that such a move provides.

6. Argument from mental causation and Meta-Problem:

- We have two hard problems, one of matter giving rise to consciousness and the other of consciousness acting back on the brain to produce ‘reports of consciousness’.
- Chalmers – “Meta Problem of Consciousness” – Provide topic-neutral explanation (independent of consciousness and its cognates) of our problem intuitions. (Chalmers, 2018)
- Meta-Challenge – confronts consciousness realists with the ‘coincidence problem’. If my belief in X is independent of X, then it is a lucky coincidence.
- The Meta-Challenge puts enough pressure on current theories of mind to cause Chalmers to be pessimistic about their prospects.
- The same Holographic (Meta-Correlational) theory (Chalmers, 2018) that generates space from Qbits is also a parallelistic theory that provides an attractive solution to the meta-problem and mental causation in general!

7. Argument from unity and disunity:

- The unity and disunity of consciousness are two sides of the same coin.
- Assume that consciousness is constituted like an Ads space dual to some relevant brain CFT.
- Assume that the CFT is realized on a large quantum computer with a nonlocal quantum error-correction code (NQECC).
- The question of the fusion and the ‘cutting’ of such spaces can provide new ways of considering ‘the subject addition combination’ problem of panpsychism and the ‘other minds’ problem.
- Spaces stabilized by NQECCs cannot be fused and cutting them in two degrades their local robustness. I will have more to say on that.

A quick review of what we did; we started by noticing that our cultural and religious traditions suggest that we are infinite from within and that consciousness may in some sense be infinite. I

then asked what it would take for us to harbor actual infinity. I then claimed that were consciousness constituted similarly to the continuum and space then perhaps we could be said to harbor a form of infinity.

While that would demand an exceedingly strange physical scenario, I suggested that modern physics has a way out and that holographic correspondence is ideally suited to turn quantum information into continuous space. I ended by claiming that this strange physical scenario has many philosophical advantages and that suggesting that we are infinite on the inside is just one of them.

Another connection between AdS/CFT, consciousness and infinity, has to do with the way that QFT harnesses infinity by using a renormalization processes. Renormalization somehow captures the invariant features of a system, regardless of extreme changes in scale. In AdS/CFT which is scale invariant, the action of the renormalization group that tames the high energy components of CFT is dual to the formation of the extra spatial dimension in the AdS. That is, the dual of the infinity taming renormalization in the CFT is an ‘Infinity generating’ process in the AdS, if the extra spatial dimension is to be continuous. This may sound strange, but Duality relations and the Holographic Correspondence are known to relate the local to the global and the fundamental to the composite.

ETHICAL CONSIDERATIONS

Not much has been written by philosophers of mind on infinity. Unlike religion and philosophers of old, like Plato, current philosophy completely separates our ethics from our metaphysics. A notable exception is Emanuel Levinas (Braungardt, 2025), who insisted that we recognize our own infinite nature by recognizing the infinite nature of the other (face to face), which for him was both a metaphysical necessity and an ethical imperative. Levinas said that – ‘consciousness of time is the time of consciousness’ (Drabinski, 2001) (Vassilicos, 2003), and as a 4-dimensionalist, you can also say that ‘the consciousness of space is the space of consciousness’ (phenomenal field, the empty room), as we said, such a space would have to possess certain properties, like:

- a. Every element of the space must have a special relation to other members of the space that it does not share with elements not in that space.
- b. It must serve as a medium enabling the robust and stable realization of information. (vacuum in QFT and elementary particles, many body ground state in Condensed Matter and quazi-particles).
- c. It would have to be finite from without and infinite from within.

Gazing into Infinity



Figure 8. My Face by Hanna Levi, 1971.
Collection: Uziel Awret.

Of course, Levinas' 'infinity' is related to his theoretical matrix and may have little in common with the AdS space that I am trying to promote here, even though its infinite on the inside.

Still, we are left with an important ethical question. 'Is it possible that the reason that conscious beings should be treated ethically is because they are infinite beings?'

As Niikawa and Kriegel suggested in their talk, "eliminativists like Kammerer" that defend eliminativist ethics need, among other things, to provide topic-neutral explanations not only for the experience of the awe of the uncontainable and our powerful Archimedean intuition but also provide topic-neutral explanations of our experience of the unity of consciousness and the powerful 'other minds problem' intuition.

A word of caution, there is something attractive about possessing an infinite nature, which means that we should take extra care when evaluating the prospects of this possibility but, in the spirit of Mel Brooks - 'It's good to be infinite' 😊.

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