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# Space-time Geometry Translated into the Hegelian and Intuitionist Systems

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## ABSTRACT

Kant noted the importance of spatial and temporal intuitions (synthetics) in geometric reasoning, but intuitions lend themselves to different interpretations and a more solid grounding may be sought in formality. In mathematics David Hilbert defended formality, while L. E. J. Brouwer cited intuitions that remain unencompassed by formality. In this paper, the conflict between formality and intuition is again investigated, and it is found to impact on our interpretations of space-time as translated into the language of geometry. It is argued that that language as a formal system works because of an auxiliary innateness that carries sentience, or feeling. Therefore, the formality is necessarily incomplete as sentience is beyond its reach. Specifically, it is argued that sentience is covertly connected to space-time geometry when axioms of congruency are stipulated, essentially hiding in the formality what is sense-certain. Accordingly, geometry is constructed from primitive intuitions represented by one-pointedness and route-invariance. Geometry is recognized as a two-sided language that permitted a Hegelian passage from Euclidean geometry to Riemannian geometry. The concepts of general relativity, quantum mechanics and entropy-irreversibility are found to be the consequences of linguistic type reasoning, and perceived conflicts (e.g., the puzzle of quantum gravity) are conflicts only within formal linguistic systems. Therefore, the conflicts do not survive beyond the synthetics because what is felt relates to inexplicable feeling, and because the question of synthesis returns only to Hegel's absolute Notion.

**Key Words:** dialectical, emotion, entropy, Euclidean geometry, feeling, formality, general relativity, intuition, language, path integrals, quantum mechanics, tensor, Riemannian geometry, transcendental aesthetic.

## 1. Introduction

Walter P. Van Stigt (see Mancosu 1988, pages 1-22) writes of the heated conflict between Brouwer's intuitionist mathematics and Hilbert's formalistic interpretation. Brouwer found mathematics to be a felt construction that grows out of a temporal intuition that represents itself in his two acts of intuition. Truth is discovered with the sense-certainties offered by the (particular) mathematical experience. The language of mathematics suffices to permit this passage, but truth is not hard-wired into its formality. Alternatively, Hilbert wanted to distill mathematics to the essential axioms, and to build mathematical proofs from a language that offered no remainder. Kurt Gödel's Incompleteness Theorem placed a firm limit on the scope of Hilbert's formalistic project. And Brouwer's intuitionist program became mired in its own demand for complex expression, negating the simplicity offered by an approach based on innate intuition.

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In this paper I argue for an approach that takes a middle way, between the extremes of Brouwer and Hilbert. Language is essential for the expression of intuition. However, language must be rich enough to encompass its own negation; it must permit its own defeat while letting our feelings escape because they are auxillary to words. Therefore, to be meaningful, symbols and expressions must be *felt*, and the feeling is necessarily beyond the scope of any formal system.

The intuitionist approach is to actively construct revelations from first intuitions, and in this paper I will construct geometry by navigation. This construction, or navigation, cannot be separated from the Brouwer`s "creating subject." Therefore, representing the space-time fabric as geometry also comes with a caveat that Kant`s thing-in-itself remains beyond appearance. Moreover, a free agent that is carried in a self-propelled vehicle will necessary construct a space-time mapping that is also two-sided. Likewise, in constructing an intuitionist geometry it would seem to be the case that the construction is necessarily two-sided as the geometrician imparts the two-sided quality onto his, or her, geometrical creation. It is this two-sidedness that relates to Hegel's system.

Hegel's dialectical logic is described in the *Science of Logic*. Innate intuition is found supporting the dialectical, even pointing to something beyond human-made words as noted in Hegel's *Philosophy of Nature*. My goal is to recast space-time geometry, bringing it in line with both formality and intuition. I hope to describe space-time as an emotive field, but this will not be a simple reinvention of a sacred geomety decribed by Skinner (2006). The intuitionist geometry will relate to innate feelings, and there will be room for the beauty and awe that is typical of sacred geometry. However, there will be no geometric abstractions raised into a Platoic world of ideal forms that exist independent of Brouwer`s "creating subject."

I must first describe some preliminaries. Hegel's logic is sometimes described as the transitions from thesis to antithesis, and finally from antithesis to synthesis. This account is often criticized for being too simplistic. A better treatment is detailed by transitions of an imperative that is first found in-itself (as objectively caricatured) before it becomes for-itself (subjectively motivated) and as the imperative finally arriving at the synthesis (the state of being in-itself and for-itself).

Following the simplistic treatment of thesis, antithesis, and synthesis, it appears that logic may start with an imperative that pretends to be in-itself (the thesis). And an intuitionist will discover the felt emotion of euphoria that is projected which stands in contradiction to the stated objectivity. Nevertheless, the imperative extends itself, and claims more ground in its fury. Therefore, elaboration must source a vitalistic feeling. However, the day will come when the imperative finds opposition in its felt other (the antithesis). The opposition will be weak at first. Nevertheless, the antithesis expresses a growing irritation, to the point that the thesis finds its defeat in a first negation: the felt imperative transforms from being in-itself to being for-itself because it can no longer deny its own subjectivity. The antithesis with its newly discovered dualism extends beyond the territory covered by the original thesis. With the first negation the irritations are relieved, but later they will reappear because the newly realized dualism is deeply conflicted with the apparent objectivity that remains. Eventually, the antithesis is itself defeated in a second negation. And with this negation a new euphoria is discovered in the synthesis achieved by combining the thesis and its antithesis.

The above logic is only a simplistic depiction of Hegel`s logic. Moreover, from an intuitionist perspective, this simple logic is too combative with thesis conflicted with its antithesis; with the pursuit of human knowledge going uncontested even in the wake of injury. In fact, the above movement from thesis, to antithesis, and finally to synthesis, is given as a double oscillation involving irritation and its euphoria. Moreover, a real antithesis is a collective, and what is found in practice are the emotion-laden judgments (caricatures) that are assigned to particular hypotheses in a time sequence.

Despite these weaknesses the three-fold pattern is found to repeat beyond thesis, antithesis, and synethesis, and this is evident in Hegel`s long elaborations. At a certain point the pattern can no longer be denied. It can only be that the synthesis represents the non-dual act of self-recognition, when a euphoric thesis as content finds its supporting context in the primary irritation expressed by its antithesis. This realization is Hegel's absolute Notion (or Concept), and to ask further questions is to return to the Notion with deeper issues.

The Notion and the flow of Hegel`s logic can now be described more succinctly: the first negation is Aristotle excluding the middle term from classical reason, and this is found offending our qualitative sensibilities. In the second negation, the irritation is found healing itself as self-awareness coincides with the tension returning to source; irritation is transformed into its other, the euphoric freedom that reintroduces of the middle term into reason. The Hegelian synthesis is the realization of a two-sided oscillation: of doubt and hopefulness. Caricatures are found in the present moment coming with meanings that source Husserl's (2001) passive synthesis, in spite of and because of the desperation projected by formality or literalism found in active synthesis.

My campaign to describe space-time geometry in Hegelian terms is dogged by the same complexities that confronted Brouwer and Hilbert. Issues surrounding the translation of Hegel out of German can be vexatious, but it may be that the complexity of Hegel's texts tends towards obfuscation even in the original language! The issue is emotional, and we must in the end be willing to admit that our own theories are incomplete. We strive for the answer that explains Hegel's Notion, but it must be ultimately accepted that the Notion is just fundamental and therefore it has no formal reason for its existence. The Notion stands in its felt starkness, and that is about all that can be said.

In section 2.1, I describe historical developments in geometry. I start with the development of Euclidean geometry, before touching on the subject of tensors, and then I move beyond to Riemannian geometry. It is necessary for us to reestablish geometry as a language, in addition to appreciating the feelings that are projected by this language. In section 2.2, I describe Kant's transcendental intuitions that are found to support developments in geometry. In section 2.3, I describe the opposition that Hegel felt towards abstractions, including geometrical abstractions that are seen as lost intuitions that have separated themselves from dialectical language.

In section 3, I argue for geometry to be seen as an intuitive construction. Accepting this establishes the connection to feeling. I also establish geometry as a two-sided language, meaning that geometry is found constructed from a logic that interacted with an emotive middle-term.

In section 4, feeling and language are used in a joint framework to unpack application areas: in general relativity, Feynman's path integrals and the mystery of quantum gravity, and in the question of entropy irreversibility. The problems surrounding Hegel's Notion are to be found even in physics.

## 2. Some Background

### 2.1 Geometry

#### 2.1.1 Euclidean Geometry

Euclid is the name given to the Greek mathematician who lived around 300 BC and authored the *13 Elements* – assuming that this author was just one person. In Book I of the *Elements*, the axiomatic foundation of plane geometry is laid (see Artmann, chapter 4). Central to this construction are five axioms having to do with a line that connects two points, a straight line, a circle, a right angle and the famous axiom of parallels. Angles are measured in degrees, and so it was also necessary to introduce definitions for *between-ness* and *order*. In this way, an angle α smaller than angle ς could be described as α<ς. Moreover, a point between two points on a line could also be described, and points could be ordered on the line.

What is found *congruent* is what looks the same, and this notion relates to geometric axioms as well as Aristotle's *principle of identity*. Aristotle's symbol "A" indicates a caricature, that finds a trivial tautology with itself, denoted by A=A. However, beyond abstraction the symbol "=" is found implying a relation, and a relation may conceal a middle term that indicates Kant's synthetic a-priori. What holds an equation together is a synthetic a-priori when the equation is a law of nature that had been first conceived and latter empirically verified; law are discovered with sense-certainties that are beyond law as caricature. What is found sense-certain points to necessary conditions that came before the caricature. Ironically, Aristotle's *principle of excluded middle* is found removing the synthetic from reason, and therefore axioms of congruency hold a potentiality that must be emphasize even within abstract geometry. An intuitionist geometry depends on the experiential as provided by the middle-term that is recognized.

The notion of congruence implies that a line, or an angle, even a circle, can be moved in space and superimposed on itself. Two geometric objects are the same if this movement is possible. Moreover, complex geometric objects can be constructed from simple building blocks. A line AB is made, an angle α attached to its end, and a second line BC is extended in the new direction congruent with the angle. On a flat surface, an angle of α degrees can be produced either to the right or to the left. Nevertheless, if congruence is expanded to include three-dimensional rotations, in addition to two-dimensional superimpositions, then all angles of degree α look the same. All lines that can be superimposed on AB look the same, all such constructions of AB with angle α attached look the same, and finally when BC is attached to the angle this completed object looks the same as all such constructions. Construction extends the notion of congruence by mathematical induction, though in Euclid's day this operation had yet to be formulated. The geometrical construction led to an equivalence class containing all such objects that look the same. As the construction was uniquely specified by line AB, angle α, and line BC, this reiterates Euclid's congruence proposition for triangles (what Artmann calls the side-angle-side theorem on page 21). What looks the same is constructed locally, but finds agreement globally. Nevertheless, what looks the same is permitted its complexification, because something remains that has the freedom to not look the same.

Euclid's derivations are highly visual. They appeal to our visual sensibilities to the extent that Kant noticed that spatial intuitions are a-priori to subsequent syntheses, emerging from a synthesis involving reason and empiricism (see section 2.2). It is remarkable that visual reasoning can be reduced to a logical formality of relation to such an extent that the a-priori intuition becomes buried (see Reichenback 1958, section 14). The axioms of Euclidean geometry when expressed in logical relations are found to be mutually consistent (see Nagel and Newman 1986, chapter II; Hilbert 1971, chapter 2), one of the few formal systems that escape Gödel's incompleteness theorem. Nevertheless, the reduction of geometry to logical relations misses the importance of innate intuition in human reasoning while implying that intuition can be distilled into formality when it can't. Perhaps it is only the formality that is supported by intuition that can make claims to consistency and completion?

If length, area and volume are going to find meaning within geometry then the concept of magnitude becomes fundamental, and with this opening new metric axioms are established. Some of these are first described in Book V of Euclid's *Elements* (see Artmann, 1991, chapter 14). Weyl's (1952, chapter 1) affine geometry is constructed from the axioms that characterize congruence by translations, and it finds itself restricted just like plane geometry. Weyl extends beyond affine geometry by adding metric axioms too. Metric functions, that measure such things as distance, are understood to be part of the axiomatic foundation of modern Euclidean geometry. Hilbert (1971) provides a comprehensive axiomatic foundation for geometry that has survived to the present time. However, Brouwer questioned the immutability of grounding axioms due to their stated arbitrariness that source an entity from language and thus find themselves far removed from intuitionist mathematics. For example, to declare that the real line is continuous by definition is to ignore the realization that continuity and discreteness are not reducible (see Van Atten 2004, chapter 3). A continuous line can be constructed by intuitionist mathematics that is not restricted to language: what is needed is Brouwer's choice sequence that engages a comprehending subject. Weyl (see Mancosu 1998, pages 93 to 101) refers to the continuum as a "medium of free becoming." A compromise is reached by noting that axioms are mathematical entities that have been created, and are permitted by Brouwer's second act of intuition (e.g., describing the demand of congruence). The axioms become spatial intuitions leaving Brouwer's first act of intuition as a temporal one. Brouwer's temporal intuition slips away from this geometry, yet in order for any language to function properly this escaping intuition must be something that is starkly felt, as required by Brouwer. We are left with axioms that can be felt or, stated another way, with axioms that cry out for empirical verification.

### 2.1.2 Tensors

Tensors find their beginning in Euclidean geometry; they are retained as necessary components of Riemannian geometry. Tensors are highly complex expressions of human intuition, pushing the limits of comprehension. The word "tensor" implies a connection to the root-word "tension", which can only be described in terms of innate feeling if we are to leave open possibilities beyond abstract caricature. Such tensors-as-feeling must necessarily provide a two-sided avenue otherwise the abstract caricature would extinguish the innate feeling upon which it rests.

The simplest tensor, the scalar, is said to be of order zero. It is a single quality that looks the same from all points of view, where points of view can change depending on coordinate transformations. Therefore, in tensors (even beyond order zero) we see an extension of congruency applied over all permissible variations offered by the geometry. If volume is to indicate a metric that finds local and global agreement, then there must be a way to describe volume as a zero-order tensor given all the various ways coordinates can be represented by geometry.

The vector is the next more complex tensor above the scalar. The vector has order 1. A vector, however, can be of two distinct varieties; it can transform covariantly or contravariantly given a transformation of coordinates offered by geometry. The geometry supplies coordinates to represent a vector **x**. The vector stands in relation to a basis, another collection of column vectors $\mathbf{b}_1$, $\mathbf{b}_2$, ... $\mathbf{b}_n$. And the coordinates are given by the contravariant components $c_1$, $c_2$, ... $c_n$, where:

$$\mathbf{x}=c_1\mathbf{b}_1 + c_2\mathbf{b}_2 + .... + c_n\mathbf{b}_n = \mathbf{Bc} \text{ (matrix and vector multiplication)}$$

Stacking the basis vectors into a matrix **B**, and transforming these into a new set of basis vectors $\mathbf{B}'=\mathbf{BA}$ (by matrix multiplication), automatically transforms the components by the formula $\mathbf{c}'=\mathbf{A}^{-1}\mathbf{c}$ where **c** is a column vector containing $c_1$, $c_2$, ... $c_n$, and $\mathbf{c}'$ is a vector of transformed components. The indicated transformation is linear (**BA**), but it need not be so. Nevertheless, this is enough to indicate a contravariant relationship, going from **c** to $\mathbf{c}'$. The vector **x** can also be represented by the reciprocal basis $\mathbf{Q} = \{\mathbf{q}_1, \mathbf{q}_2, .... \mathbf{q}_n\}$, where $\mathbf{Q}^T=\mathbf{B}^{-1}$, and this gives the covariant components, a column vector **d** containing $d_1$, $d_2$, ... $d_n$ where:

$$\mathbf{x}=d_1\mathbf{q}_1 + d_2\mathbf{q}_2 + .... + d_n\mathbf{q}_n = \mathbf{Qd}$$

The transformation **BA** also transforms **d** to $\mathbf{d}'$ by the covariant relation $\mathbf{d}'=\mathbf{A}^T\mathbf{d}$. A first order tensor will either transform covariantly or contravariantly, and a covariant (or contravariant) vector will always remain a covariant (or contravariant) vector after the coordinate transformation. The form (covariant or contravariant) of a vector is an invariant, providing another congruency. The contraction of covariant and contravariant components, given by

$$d_1c_1 + d_2c_2 + .... + d_nc_n = \mathbf{d}^T\mathbf{c} \text{ (vector multiplication)}$$

is also an invariant scalar (tensor of order 0). And, provided **x** is a vector in Euclidean geometry, this particular contraction gives the square magnitude of **x** as something that looks the same for all coordinate transformations.

Tensors can be of any order, where each level of the order permits either a covariant or contravariant transformation of components. A second order tensor is like a square matrix, with two types of components. In tensor notation, superscripts are usually reserved for the contravariant class while subscripts are reserved for the covariant class. Therefore, $T^v{}_u$ is a second order tensor representing contravariant component v and covariant component u. Tensor contractions also generalize, by setting any contravariant class equal to any covariant class, and summing over the dimension where components vary within the selected class. In general this reduces a tensor of order N to a tensor of order N–2. A tensor of order M can be multiplied to a tensor of order N to produce a tensor of order M + N.

This level of algebraic book-keeping is necessary to keep track of congruency while one coordinate system is transformed into another. A good example is provided by a scalar function f(**y**) that is said to look the same for all coordinate systems that transform **y**. The function f(**y**) can be represented by an infinite tensor series:

$$f(\mathbf{y}+d\mathbf{y}) = f(\mathbf{y}) + \Sigma_i T_i dy^i + ½\Sigma_{ij} T_{ij}\, dy^i dy^j + .....$$

where higher order covariant tensors ,$T_i$, $T_{ij}$, …, lurk beneath the specification of f(**y**) as a scalar fuction, and the differentials, $dy^i$, $dy^j$, … , act as contravariant tensors. Each term in the series is found to be an invariant scalar found by contraction.This series is not the normal Taylor series found by differentiating f(**y**), but this series is found agreeing with the Taylor series term for term as the differentials, $dy^i$, tend to zero. The fact is that the covariant tensors represented in this series are underdetermined given f(**y**) and its derivatives.

As tensors present themselves contravariantly and covariantly, the above tensor series can be re-expressed in terms of the contravariant components $T^i$, $T^{ij}$, … and the covariant components $dy_i$, $dy_j$, … :

$$f(\mathbf{y}+d\mathbf{y}) = f(\mathbf{y}) + \Sigma_i T^i dy_i + ½\Sigma_{ij} T^{ij}\, dy_i dy_j + .....$$

The two tensor series represent different versions of the same series, the only difference is the emphasize given to contravariant and covariant components. To emphasize the contravariant is to emphasize a direct accessment of n basis vectors. To emphasize the covariant is to emphasize an indirection accessment of basis vectors: replacing a vector with the orthoganl projection formed by the remaining n-1 vectors, and doing this in turn for each of n vectors. This implies that $T_{ij}$ and $T^{ij}$ represent the same second order tensor expressed differently, among the other tensors that are also present.

Starting from any point, it is possible to navigate the geometry by following a map that moves an amount that agrees with either the contravariant or covariant components by accessing the basis vectors accordingly. This navigation is provided by an intuitionist construction. However, the logical map so generated tends to be extrensic, and it can miss the local details that are recovered by returning to the intrensic surface features. To find an invariant it becomes necessary to contract tensors again, bringing together both contravariant and covariant componets. The map need not agree with the territory, and so geometery is found two-sided.

That which underlies the two-sided formality is sufficient to support our human understanding, which is beyond the formality. It is interesting that a tensor may represent something that is sense-certain, for example stress. The quantitative is merely translated into the formality provided by geometry, but because the tensor is two-sided the qualitative distinctions are fully sublated in the stipulated congruency. The beauty we feel when looking at a tensor equation is our own, this quality has escaped from the tensor formality.

### 2.1.3 Riemannian Geometry

Euclid's fifth postulate, the axiom of parallels, leads to the flat Euclidian geometry. A version of the fifth postulate states that for any line *l,* and a point A removed from *l*, there is a unique line *l´* through A that is parallel to *l* (Greenberg 1974, pages 18 to 20). Euclidian parallel lines never intersect. Greenberg tells us that the axiom of parallels was never readily accepted among mathematicians. Part of the problem is that this axiom cannot be constructed from primitive intuitions that emerge from experience. Greenberg writes that the fifth postulate is different from the other axioms, because "we cannot verify empirically whether two lines meet, since we can draw only (finite) segments, not lines." We are forced to justify the axiom of parallel indirectly, for example by verifying that the angles of a triangle add to 180 degrees. However, Euclid's interpretation of geometry was discovered to be non-privileged, as elliptic and hyperbolic geometries were discovered once the fifth postulate was relaxed.

What are needed for geometrical representation are grid lines that are somehow constructed from primitives. This need is studied again in Section 3, but it suffices to point out that the flat Euclidean geometry was not special beyond its flat grid lines. Moving to the most sophisticated treatment of differential geometry is where Riemannian geometry enters (developed by Gauss, Riemann, Minding and many others). Grid lines correspond to the components **x** representing a vector that points to a location in space. But this vector is now a tensor, and we may stipulate that the components **x** transforms contravariantly. Moving beyond the coordinates (components) that might as easily correspond to the Euclidian grid we find not the flat surface at point **x**, but an innate curvature. The curvature may be denoted by $g_{uv}$, a second order tensor that transforms covariantly, called the fundamental metric tensor. In Riemannian geometry the fundamental metric tensor is rarely made redundant given the magnitudes expressed along grid line. Rather length is measured by integrating over a contraction, $|\Sigma_{uv} g_{uv} dx^u dx^v|^{½}$, from location to location. The locating grid lines and the fundamental metric tensor represent a two-sided geometry.

### 2.2 Kant's Transcendental Aesthetic

Kant (see Meiklejohn 1990, pages 21-43) noted that the form of space indicates an extension beyond our self and provides a representation of an external three-dimensional reality. Points removed could coexist in space. Moreover, this spatial form comes before experience, i.e. as an a-priori intuition that supports experience.

Kant also noted that the form of time indicates an internal dimension. Unlike space, time unfolds in one dimension. It represents a succession of prior points, and a future yet to be navigated. While the self feels a sense of permanence, time points do not coexist like spatial points. The temporal form comes before experience and so it too comes as an a-priori intuition that supports experience.

Space and time are different intuitive forms, but they are surely interrelated. The time intuition grounds what coexists in space, as surely as the space intuition grounds what changes as a singular progression. The thing-in-itself remains beyond our spatial and temporal intuitions. But these intuitions are pure forms.

Intuitions are discovered as syntheses, as much as agreements between reason and empiricism, and these can exist in thought. These pure thoughts can give their support to mathematics and abstract geometry. In Kant's day, Euclidean geometry was believed to be the obviously correct geometry that had abstracted truth from physical space. However, it is a misconception to see Kant's "transcendental aesthetic" as an unqualified endorsement of Euclidean geometry, despite the many claims to the contrary (Randall 1998, Palmquist 2001). At best, Kant describes an a-priori, or a transcendental geometry, that need not stop at Euclidean geometry as a pure form. The fact that today Riemannian geometry has replaced Euclidean geometry only validates Kant's treatment of the a-priori: the intuitions that science is able to refine are again found as a-priori conditions emerging from the synthesis of reason and empiricism. Moreover, Euclidean geometry is only well represented by the spatial dimensions where extension is felt. The time dimension indicates a unidirectional succession that is not reversible. The transcendental geometry that had been felt a-priori was already very different to a four-dimensional Euclidean Geometry, even in Kant's day. Nevertheless, section 3 shows how Euclidean geometry as a pure form can serve as an extrinsic component to an otherwise rich space-time geometry. The irony is that Einstein was able to unify time and space in special relativity, but section 3 shows that we still ended up with a two-sided geometry indicating a synthesis of both the intrinsic and the extrinsic.

## 2.3 Hegel's Objection

Hegel reacted against a pure intuition that exalted itself by the quantitative extensions that are typical of mathematics. These expressions tend to complexity, and Hegel was critical of complex searches that are unable to feel their qualitative origin (see Miller 1969, page 228) . The one-sided expression that goes on and on is only Hegel's "spurious infinite" that keeps repeating its self while never discovering anything new. Hegel writes that "the hollowness of this exaltation, which, in scaling the ladder of the quantitative, still remains subjective, finds expression in its own admission of the futility of its efforts to get nearer to the infinite goal, the attainment of which must, indeed, be achieved by quite a different method," (Miller, page 229). The extension of an axiomatic system is post-synthetic, but what gives meaning to the grounding axioms provides a qualitative distinction that tends to get ignored given the complex expression that can grow out of the grounding axioms. Hegel was interested in excavating the qualitative; he wanted to dig deeply into Kant's synthetic a-priori to approach the thing-in-its-self. He disagreed with Kant, believing that it was possible to see and reason beyond the synthesis – through a dialectical logic that respected the union of opposites and permitted a passage into the transcendental. Hegel's logic has no axiomatic beginning: what is there has to be discovered.

Hegel's weakness was his aversion to abstract mathematics, including geometry. Kant's transcendental geometry was one-sided, unable to see beyond its grounding axioms. Hegel writes that "axioms [...] considered in and for themselves, require proof as much as definitions and divisions, and the only reason they are not made into theorems is that, as relatively first for a certain standpoint, they are assumed as presuppositions," (Miller, page 808). The dialectical passage could not be found in Euclidean geometry, and what is required is nothing less than Hegel's Notion. There was no union of opposites in geometry, Hegel (Miller, pages 813-814) writes: "It is only because the space of geometry is abstraction and void of asunderness that it is possible for figures to be inscribed in the indeterminateness of that space in such a manner that their determination remain in fixed repose outside another and posses no immanent transition into an opposite." These beliefs were Hegel's mistakes, but in fairness Hegel`s objections were raised against formalistic mathematics and not the intuitionist version presented here. If anything, an opposite finds itself in its own reflection, and reflection reaches its highest expression within the structure provided by a projective geometry invented by an intuitionist.

Euclidean geometry was to pass over into Riemannian geometry, but Hegel only vaguely anticipated this future event. He had hardened his heart against mathematics, and he could not understand that geometry was itself a language. Kant's a-priori intuition was still there, looking for its reflection in a way that Hegel could appreciate. Hegel writes that " the so-called explanations and the proof of the concrete brought into theorems turns out to be partly a tautology, partly a derangement of the true relationship, and further, too, a derangement that served to conceal the deception practiced here by cognition, which has taken up empirical data one-sidedly, and only by doing so has been able to obtain its simple definitions and principles", (Miller, page 815). The beauty of geometrical intuition had to wait, even as Marx, Engels and Lenin acknowledged the dialectical nature of mathematics (see Kol'man and Yanovskaya, 1931). Marxist materialism wanted to extinguish the strong intuitions felt by the likes of Weyl and Brouwer. Nevertheless, the quantitative is to pass over into the qualitative, and the Divine is to spring from geometry as the sense-certain intuition is felt again. Freedom is discovered even in the one-sided drive to extinguish intuition as permitted by Hegel's first negation. The second negation turns Marx on his head and returns Hegel to his upright position.

## 3. The Hegelian Synthesis

### 3.1 Route Invariance and One-pointedness

*Duration* and *distance* cannot be defined separately: duration is a distance traveled by light; distance is a duration required by light (i.e., given the discoveries of special relativity). *Straightness* is a one-pointed extension given by one unit of distance, an extension that leaves no shadow in a perpendicular space. *Curvature* is an adjustment in orientation that is not indicated by prior movements involving duration, distance and straightness. A constructive treatment of these definitions will be presented in section 3.2.

A *rigid body* acts as a ruler that measures distance, and can be transported to its end and extended in a one-pointed direction. The rigid body defines endpoints *simultaneously*, and only through such declaration can time be sublated in space. This provides a method to measure length while giving the false impression that duration is not involved. The record of each extension can be collected into a single vector to provide the coordinates of an endpoint denoted by A. The movement that progresses from the *origin* to A, and is measured by rigid body placements, produces the coordinates as a tabulation of the placement information (the ruler's length and its directions given that the ruler has the property of *one-pointedness*). One-pointedness indicates simple directionality, the ability to point at something, and it will be discovered related to the property of *route-invariance*.

The nature of the placement information comes into question. The agent that moves the ruler from position to position, the same agent that records the placement information, might also hold the abstract understanding of Euclidean geometry. This abstract understanding of geometry will suggest how the placement information is to be processed. The coordinates of A are given by $\mathbf{v}_1+\mathbf{v}_2+...\mathbf{v}_n$, where $\mathbf{v}_i \in R^N$ (N=3 from observation) is the adjusted placement of the ruler at position i-1, $|\mathbf{v}_i|$ is the ruler's length, and $\mathbf{v}_i$ is pointed in a direction that may differ from $\mathbf{v}_{i-1}$ depending on the curvature perceived at placement i-1 (i=1, 2, ... n).

The ruler placements, and the ensuing vector additions, must lead to unique coordinates of A if this calculation scheme is to make any sense. Yet the ruler that moves from the origin to A can proceed by many routes. This assumption is easily confirmed on our planet earth. The vector additions going from the origin to A can proceed by many different paths, into deep valleys and up tall mountains, or even by the shortest path that is available to birds. It matters not which path is selected in our ruler placements, the same coordinates for A are retrieved each time without error. This invariance property is not implied by the definitions of duration, distance, straightness, and curvature. Something else beyond these definitions is behind this property if we are to progress beyond Euclidean geometry. Moreover, the vectors that go into the summation have no necessary order. Vector addition is a communicative operation, giving the same answer independent of the order of addition. Adding the vectors in reverse only signifies a different path to A, even if the ruler disappears and finds passage well beyond our normal space-time perceptions and then magically reappears at A.

Route-invariance is a property of a coordinate system, within which information contained in the coordinates that permit the passage from the origin to point A is invariant to the particular path to be taken. This information is teleological as it relates to a goal, and it is contextual as it is opposed to one-pointedness. If at the (i-1)-th step a ruler's orientation sensed from distance, straightness and curvature is encoded into a vector $\mathbf{v}_i \in R^N$, then route-invariance in Euclidean geometry implies $\mathbf{v}_1+\mathbf{v}_2+...\mathbf{v}_n = \mathbf{w}$ is a constant for all paths starting at the origin and ending at point A with coordinates $\mathbf{w} \in R^N$. To accommodate time as understood from special relativity, the measuring paths are required to share a common proper time in that their departure from the origin is jointly coincident while their arrival at A is jointly coincident; or these occurrences can be assumed to be coincident to good approximation. Even if space is innately of higher dimension beyond three, and the incomplete specifications given by $\mathbf{v}_i \in R^3$ (i=1, 2, ... n) are only higher dimensional projections onto an imagined 3-D manifold, the route-invariance property is still active as seen in the incomplete projections. Letting the ruler's length shrink to zero preserves the route-invariance property for general curves.

The route-invariance property is a necessary condition if a coordinate system is to have logical meaning. The coordinates of A provide the logical signature of going from the origin to A. Route-invariance is a condition of logical passage assuming I am pointing to A on a map and asking a taxi to take me to A. More generally, this condition is necessary for a grid, otherwise geometric representation is not possible. But note that this grid is *extrinsic*, and it therefore says little about the terrain and landscape that is to be discovered going from the origin to A. The landscape is the *intrinsic* geometry where curvatures are to be discovered relative to associated grid-points. Duration, distance and straightness are also re-defined at the grid-points making local and global agreements. The taxi driver might take me for a real ride from origin to point A – including extra travel time and a concomitantly excessive charge on my credit card!

There is a tendency to emphasize geometrical invariance and to reduce geometry to the intrinsic surface, even to excommunicate the extrinsic from science and mathematics. An extrinsic coordinate system is not uniquely determined. However, a curved intrinsic surface stands in relation to a flat extrinsic geometry. In the absence of an extrinsic geometry where route-invariance clearly emerges from first principles one must question what curvature means. Intrinsic route-invariance ought to be transparent, but what we generally discover are obtuse properties that are smuggled in without mention.

The extrinsic grid can remain flat to protect Euclidean route-invariance. An extrinsic direction can always be related to a direction along a unique geodesic that is confined to the intrinsic surface. Moreover, the entire extrinsic basis forms a set of independent directions that can be *parallel transported* from grid-point to grid-point along geodesics. The transported direction set reflects how the directions have turned, thereby projecting the extrinsic grid onto the intrinsic surface. This relation between the extrinsic and intrinsic cannot be severed. In general, intrinsic grid-lines will not protect an Euclidean route-invariance, and without some kind of route-invariance the coordinates are hardly meaningful due to the passage from origin to point A becoming undetermined (i.e., if route-finding determinations are to emerge only from primitive definitions such as duration, distance and straightness, and not smuggled in covertly). Extrinsic route-invariance implies that no two parallel grid-lines can cross; yet it may happen that the projected grid-lines on the intrinsic surface may cross (e.g., two geodesics can cross). In this situation, two (or more) extrinsic coordinates correspond to one point on the intrinsic surface. The extrinsic grid is well matched with the intrinsic surface when a one-to-one mapping is maintained; otherwise such operations as surface integration become a challenge. Extrinsic route-invariance is an a-priori assumption whenever a coordinate is mentioned. The coordinates must contain enough information to permit one-pointed passages from the origin to a point removed, and the information must be transparently obtainable from first principles.

### 3.2 Geometrical Construction

Distance, straightness and curvature are defined as constructive measurements that respect route-invariance, and are based on no further differentiation provided by an exterior agency. To measure is to engage the synthesis of reason and empiricism, permitting an escape beyond Euclidean geometry with its implied abstractions. To measure duration we measure distance, the distance a clock's hands have moved or the distance traveled by light. Likewise, to measure distance we measure duration, the amount of time required for something to move between two points. To hide time in pure distance and the endpoints of a rigid body are declared simultaneously. We can add distances together, and we can subtract distances to retreat to a prior position where simultaneity is recognized with rigid bodies. Our experiences with duration are different as it has been possible to add too duration while subtraction is forbidden; i.e., we don't experience time reversal. The time passage is irreversible and indicates a broken symmetry. A consideration of time is a-priori to the recognition of spatial simultaneity, and more generally simultaneity is only relative to the motions found among reference frames.

We will start our construction with duration and distance; in much the same way that Hegel started his dialectical passages from being and nothingness, which were later transformed into becoming. A reality where distance and duration are experienced, and nothing else, is limited to one temporal dimension and one spatial dimension. There are in fact more dimensions, but our capacity to experience them requires something beyond the primitive sense imprints of distance and duration: it requires sense-certainty, and the synthesis of empiricism and reason to which this gives rise. Distances in higher dimensions can always be projected onto a one-dimensional geometry where route-invariance applies. Route-invariance in one dimension honors the fact that distances can be added and subtracted while preserving the meaning of simultaneous position.

One-pointed distance passes over into straightness, but sensing straightness implies the perception of a perpendicular space. This implies no less than two straight directions, $\mathbf{s}$ and $\mathbf{s}_\perp$, indicating a direction and its perpendicular other. An extension in direction $\mathbf{s}$ that goes undetected when projected onto $\mathbf{s}_\perp$ is called straight. The undetected nuance belonging to the extension that is projected into $\mathbf{s}_\perp$ is the first negation that differentiates pure distance from straightness. It is now possible to transport both $\mathbf{s}$ and $\mathbf{s}_\perp$ to the end of the extension in the direction of $\mathbf{s}$; this is the so-called parallel displacement again. A new straight extension from the new point can be made in the direction of $\mathbf{s}$ provided that its projection into $\mathbf{s}_\perp$ continues to vanish. This process of finding these straight extensions can be continued in like fashion, and the extensions can be added together to provide an overall distance on a route of travel that is called straight. By working with infinitesimal extensions, a curve can be so constructed that it is found straight. Now route-invariance applies in two dimensions, and honors the fact that distances in direction of $\mathbf{s}$ and $\mathbf{s}_\perp$can be added and subtracted while preserving the meaning of both simultaneous position and parallel displacement. The recognition of simple straightness does not imply sufficient agency to see more than two spatial dimensions. Straight distances in higher dimensions can always be projected onto a two-dimensional geometry where straightness is perceived in its primitive form.

A curve that is not straight shows simple curvature, but this negative does not imply a transition to a wider awareness in the Hegelian sense. The transition where straightness passes directly over into curvature is described by Hegel's second negation, and is provided by primary curvature. Seeing primary curvature implies that distances built from straight movements can later be shown to be non-straight; i.e., a new direction **r** is recognized that is perpendicular to both **s** and $\mathbf{s}_{\perp}$ where the extension along the direction **s** when projected into **r** does not vanish. It happens that the straightness offered by **s** and $\mathbf{s}_{\perp}$ produces a bent curve when a third dimension is later sensed more fully, as indicated by the discovery of **r**. Route-invariance now requires three spatial dimensions to protect the meaning of simultaneous position and parallel displacement, while permitting primitive definitions that appeal to our prior intuitions. Belief in a flat earth gave way to an understanding of a spherical earth due to the felt realness of the third dimension. The perception of primary curvature does not imply sufficient agency to see more than three spatial dimensions.

The system of constructive definitions (discovered locally and finding their agreements globally) requires at minimum perception in four dimensions: three spatial and one temporal. Agency is always indicated by local and global agreements, even constructions discovered by their emerging from primitive definitions that appeal to our prior intuitions. These are conditions of necessity, not sufficient conditions. The dimensions we discover are starkly real; it is sufficient that there be enough dimensionality for our self-awareness and no more, given that impetus mirrors Hegel`s Notion. What is sufficient is that any higher dimensions are projected onto the dimensions that awareness can accommodate, leaving our primitive intuition more or less intact. It cannot be that awareness of higher dimensions destroys what was learned from the first few dimensions.

The impetus that sought measurement has no where else to go in our normal understanding of time and space, as is evident by our bluntly seen three dimensions of space and one dimension of time. The impetus that is the pure understanding can see the Trinitarian archetype that signifies self-recognition and Hegel`s Notion, and so it has no more need for crude measures of time and space. This is not to say there are no more dimensions, it is only that the impetus must now be directed inward to find new dimensions as the impetus learns to self navigate further. And given our complext abilities to naviagate space-time it is clear that there are many more dimenisons beyond what is normally attributed to space and time.

Once arrogance is recognized by Hegel`s "cunning of reason," it becomes necessary to suspend the wayward activity in anticipation of arrogance`s negation. Arrogance is blinded by its emotional attachment to the extrinsic, and a new direction is sough coming from the intrinsic. I speculated that the new direction relates to the coming awareness of a higher dimension.

It is not only humans that can navigate the space-time fabric while showing great mastery. Migratory birds fly hundreds of miles, if not thousands, to overwinter in warm climates. Salmon return from the open ocean to the stream bank of their birth, to lay eggs for a new generation before dying. The Monarch butterfly has equally amazing migratory instincts. The hint is that life can navigate the space-time fabric by relating to something vital that carries intentionality (or teleology) and that is hidden within the fabric. If space-time is to be caricatured well by geometry, honesty demands a two-sided construction lest my creation be not beautiful.

Bennett (1956, Chapter 15) describes a universal geometry with a necessary six dimensions, enough to distinguish all interacting occasions that are recognized in the physical world. However, the extra dimensions probably represents an infinity that cannot be reached by reason working alone; agreeing with Nicholas of Casa (see *On Learned*

*Ignorance*). It is unlikely that a one-sided reason can find perfect agreement between the extrinsic and intrinsic, or between one-pointedness and route-invariance. The ladder of dimensionality then must engage our emotions, and so the extra dimensions are probably not the qualities predicted by typical science that restricts itself to the mechanics on Riemannian geometry. The impetus most also tame our emotions! The recognition of the Hegelian synthesis represents the passing of awareness into higher dimensions where both doubt and hopefulness can act, but this is not the activity of a one-sided reason.

### 3.3 Geometrical Language

My usage of duration, distance, and straightness were motivated because these very qualities are found necessary for a logical grid-system that facilitates the correct application of maps and coordinate systems (as discovered with Euclidean geometry). The map is not the territory, however. The map only permits navigation to retrace the intuitionist trail. The grid-system is only extrinsic to the intrinsic surface features. It remains only to engage the concrete territory and negate the extrinsic map by becoming aware of an extra dimension where a more local curvature is discovered, and where the covariant is found joined to the contravariant.

The geometry just described is the synthesis of the extrinsic and the intrinsic, and it is not necessarily the Euclidean geometry originally imaged by Kant that found its support from a-priori intuitions. It is only the extrinsic that is so constructed to be isomorphic or unconflicted with Euclidean geometry. The intrinsic may represent an unusual topology or a finite region embedded inside an Euclidean geometry, otherwise the intrinsic is implied by its curvature. The extrinsic may signify the domain of a function, $f: R^n \to \Omega$, that indicates curvature. The synthetic I have described above is more generally a Riemannian geometry. Nash (1956) showed how it was possible to embed Riemannian geometry into a higher dimensional Euclidean geometry. Morgan (1998) develops his simplified account of Riemannian geometry from manifolds embedded in Euclidean geometry. The extrinsic coordinates are the contravariant components of a tensor of order 1; the intrinsic curvatures are the covariant components of a tensor of order 2 (i.e., fundamental metric tensor, denoted by $g_{uv}$).

The two-sided behavior of Riemannian geometry is evident from the way in which the metric tensor responds to coordinate transformation. The tensor $g_{uv}$ transforms covariantly. However, at each point on the curved surface there are two local basis sets; one set transforms covariantly while the other transforms contravariantly. The contravariant basis can be extended globally and used as a replacement for the extrinsic basis. It is also possible to extend the local covariant basis into an extrinsic system, but in so doing the fundamental metric tensor becomes $g^{uv}$ and transforms contravariantly. The two sides of Riemannian geometry relate to the covariant and contravariant, and these qualities are found to be joined to the extrinsic and intrinsic, depending on how the system is structured.

In the process of fitting abstract geometry to concrete reality, we discover a-priori definitions that demand empirical verification. These are Reichenbach's (1958) "coordinative definitions", examples being duration, distance, straightness, curvature and route-invariance. To these we must add Einstein's constancy of light speed, and his equivalence of gravity and acceleration. In a limited sense, these find their support in the synthesis of reason and empiricism. The definitions become Kant's a-priori that mysteriously emerge from the synthesis. Hegel's indicates that the most fundamental entity that escapes the analytical mind is its dependence upon antecedent definitions (see Miller 1969, page 43). It is true that space and time are necessary for the framing of experience as Kant speculated. However, it is not the case that space-time geometry is sufficient to explain the relation as it is experienced. What remains is an innate intuition that Kant felt yet it escapes reanalysis by reason, as it is a-priori. Science discovers what is necessary but not sufficient, and to inquire about sufficiency is to follow our experiences beyond the normal framework provided by science.

My account preserves Kant's transcendental aesthetic as a system built from prior intuitions that source the synthesis of reason and empiricism. The temporal intuition is strongly associated with one-pointedness (the negation of route-invariance, or the feminine aspect), the spatial intuition is associated with route-invariance (the negation of one-pointedness, or the masculine aspect). The temporal and spatial intuitions signify a unity in opposites, with both indicating a raw directiveness that eventually must slip into the beyond. Kant's peers were mistaken only with the early attachment made to Euclidean geometry, his a-priori intuitions are found to support Riemannian geometry today. I have also addressed Hegel's criticism of geometry as the above system is permitted to unfold from a two-sided fabric, reflecting a dialectical quality in geometry given as a language. What is beyond one-pointedness and route-invariance is also beyond any geometry that is only post-synthetic. What is beyond is qualitative and the source of innate intuitions. Brouwer believed that language expresses will-transmission, and beneath language is raw aggression if not a more cultivated social emotion (see Mancosu 1988, pages 40-53). It is only with the human contribution that symbols and words are found married to emotions and intuitions. Smith (2007) speculates that Hegel's dialectic is a felt movement, and this can only be because a feeling is negated by the thought that circumscribes it.

## 4. Specific Examples

### 4.1 General Relativity

Point A is something that can be pointed at, but if the rigid body (as ruler) has mass and it is thrown at a target A then the ruler can miss the target. At the *ith* step, the ruler with given mass will only have a momentum that is pointed in a particular direction. The prior goal of leading to point A is already encoded into the momentum. The trajectory of the ruler will encounter other factors that will cause it to change its extrinsic direction. In this sense the target A is not a target per se, rather it is a direction that is sensed along the trajectory. And what is sensed is the dullness of a rigid body that lacks agency, having only mass. The trajectory finds modifications because of its interactions with the intrinsic geometry, which includes the curvatures that are encountered. And even the dullest activity must also maintain consistency with extrinsic route-invariance as this is the only way the Divine can take flight - qualitative distinctions are permitted to escape as they are able to hide behind quantitative characteristics provided by the geometrical language. The rigid body finds the singular path among all routes that minimizes the proper time in passage - the geodesic path, the only path that can be reflected back to the extrinsic geometry where route finding has been given meaning. The ruler's dullness can demand no more time, and is eager to take flight. This restates Einstein's equations of motion that are derived from the principle of equivalence.

Einstein's equivalence of inertia and gravity is two-sided. Moreover, Riemannian geometry is demonstrably two-sided, and we note that the middle term that holds the two-sided geometry together has escaped reason if only because Aristotle could not tolerate a logic with an emotional center. The motion of rigid bodies indicates the flux of mass that issues forward from the gravitational tensions that are somehow woven into the fabric of space and time - a motion that might give the false impression of a deterministic clockwork universe that is independent of emotion. Einstein's field equations are also needed to re-introduce gravitation as a tension to affect the intrinsic curvatures. This gravitational tension is no less than the expressed demand of extrinsic route-invariance paired with the requirement of intrinsic one-pointedness - a demand that is specified by the formality of general covariance leading to an invariant tensor equation that translates stress into curvature.

General relativity is, therefore, true by its linguistic definitions, and the definitions have been merely translated into the language of geometry. The language emerges from stipulations that have to do with invariance, affine connexion and metrical measures (Schrödinger 1950), and all of these indicate a congruency that hides a synthetic. The language acts as a conduit to convey the meaning of tension, almost as good as the actual fabric comprised by space-time. We apply tension to the balloon by deforming it, and this simple expression has as much explanatory power as the field equations. General relativity is given by the conditions of necessity that have been built into the language. General relativity is not, however, sufficient to explain its own definitions. Einstein's geometry is post-synthetic, and no language is sufficient to explain the magic that language offers. For example, it is sufficient to note that the balloon deforms because of outside agency, independent of the stretching potential of the balloon. And to question these definitions (that make up any law) is to question the middle terms that Aristotle tried to exclude from reason, terms that cannot be excluded that are also designed to escape into Hegel's Notion. This leaves general relativity with its bluntness, and with a synthetic that is shrouded in qualitative distinctions that also limit application.

Alternatively, taking Einstein`s equations to be absolute fixtures in a Platonic realm might equally lead to distortions or even fantasies, perhaps letting Kurt Gödel leap to the conclusion that time is an illusion (e.g., see Yourgrau, 2006, chapter 7). Gödel`s time did the affectionate work of creating an abstraction called the Gödel Universe, but his time was unable to see itself as a love of abstraction. It is plausible that absolute time is real but is an intuitive dimension (an "ideality" as Gödel stipulated), and is unable to be put into a strict formality like the Gödel universe. Perhaps time is what gives privilege to reference frames that can see even abstraction, and therefore its duty is to escape formality where such privilege is deemed impossible thereby fooling even Gödel? That is, the great escape is what gives birth to seeing our affection but only after our affection for abstraction is tamed? Is not the great escape enforced by the conservation of energy? Does not general relativity reflect the great escape while coincidentally mirroring the blunderbuss flight of dull masses? Do our affections make us equally dull? Minkoski's "block universe" was conceived from special relativity, and it too cannot provide a justification for an absolute determinism, contrary to many literal interpretations. Geometry must return to something two-sided, to permit our escaping intuition that is unable to give absolute privilege to abstractions that are only post-synthetic.

### 4.2 Feynman's Path Integrals and Quantum Gravity

A photon that is projected forward, from a source to a detector, can follow various paths that are described in intuitive terms by Feynman (1985). Feynman's approach is an intuitionist construction that describes the properties of vectors that find themselves in a two-dimensional Euclidean space. This construction is in the best tradition offered by

Brouwer's two acts of intuition represented by time succession and the time-sublated forks that penetrate spatial possibilities (see van Atten, 2004, page 4). Therefore, depending of the mode of intuition, vectors can be combined in one of two ways: either by vector "multiplication" or "addition".

Any vector in the two-dimensional geometry indicates both a direction and a magnitude. Magnitude translates into intensity, and because intensity also relates to frequency (color) the magnitude of any vector signifies the probability of an event occurring in the quantum realm of subatomic particles such as electrons and photons. Time is translated into space, a two-dimensional direction that indicates phase. Any event that can be traced out in three spatial dimensions and one time dimension reduces itself to these two dimensional arrows from the point of view of light. Light that feels no duration sees only two-dimensional space with its one-pointedness. This is the same information contained in a clock's hand that rotates over its history (as phase), points in a two dimensional direction and shrinks itself to reflect its own intensity.

Feynman's spatial intuition permits all possible paths from source to detector. And if the two-dimensional vectors are fixed given the context of a fork that points to different routes, the intuition of route-invariance only detects the collective state given by the quantum superposition. Two-dimensional vectors can be added, and in this way their intensities can be amplified or cancelled. The vectors among all choices offered by the fork can be added together, and route-invariance gives the same endpoint, independent of the choices. Beyond the summation all the choices look the same to route-invariance. Nevertheless, the shortest paths - the paths requiring the least time - dominate the summation, while other paths cancel. These dominant paths reflect the collective, an expression of context and teleological impetus.

Feynman's temporal intuition gives the two-dimensional vectors as a succession: instead of adding the vectors they are now "multiplied" together, with any two vectors combined into one by adding together the individual time from each vector to find the overall phase given as the new direction of the combination, and determining the magnitude of the combination (new direction) as the two magnitudes multiplied together.

Feynman's intuitions have found agreement with light and electromagnetism, and nothing else is discovered beyond these primitive forms. His approach provides an alternative to classical quantum mechanics, where wave functions can indicate states of superposition. To detect with the intuition of one-pointedness causes the wave function to collapse. A probabilistic distribution is discovered from multiplying each wave-function by its complex conjugate.

The intuition of route-invariance senses the form of future possibilities in the state of superposition. One-pointedness brings an irreversible selection to this intuition, it negates the plurality while sublating any teleological causation carried by the superposition. This switches the spatial into a temporal intuition, where vectors can be multiplied again while continuing to uphold the validity of classical probability. The broken symmetry is also presented by the wave function collapse, an action that looks to be a progression from past to future. Therefore, route-invariance implies a symmetry consideration whereas one-pointedness brings an action principle.

This only begs the question of the cause of the collapse, as the cause is still beyond Feynman's intuitions that are merely translated into a language. The collapse was caused by a presumed quantum gravity, and this is about all that can be said given that known quantum mechanics leaves the collapse under-determined. The probabilistic determination leaves much unexplained, leaving little said about the future context that somehow got transported to the past.

The charge of idealism may be leveled at my account. Nevertheless, quantum gravity and its connection to wave function collapse have already been implicated in human cognition and provide the basis for the orchestrated objective reduction hypothesized by Hameroff and Penrose (1996). My suspicion is that the cause of wave function collapse will slip away entirely from any formality, leaving Hegel's Notion in its starkness. Formality is language, and as with space-time geometry, language works because something slips away while leaving a feeling in its wake. One-pointedness and route-invariance come as a-priori intuitions that find unity in opposition, and while they are constructive they leave a middle term that carries a qualitative distinction. It is no wonder that quantum gravity remains a mystery.

### 4.3 Entropy Irreversibility

Physical laws, like those described above for general relativity and quantum mechanics, hold symmetry properties. They describe action principles that look the same from all points of view offered by any particular symmetry, and in all cases this action looks the same even if time runs backward. Laws derived from symmetry considerations cannot account for the asymmetries that show themselves in time. The asymmetries that reveal themselves are beyond these laws, and so it is here that the second law of thermodynamics intervenes and proclaims that entropy, or disorder, must increase with time. Perhaps symmetry merely points to the weakness of abstraction. Perhaps it is only symmetry that permits the reissuing of laws by subjecting them to Hegel's second negation, while weakening the laws enough to permit the Spirit's escape. Sections 4.1 and 4.2 reduced laws to conditions of necessity that follow in the wake of the escape, but this leaves the laws insufficient to explain Hegel's Notion. But can Hegel's Notion survive the second law?

It has been found that attempts to explain the asymmetries from symmetries were not tenable. Ludwig Boltzmann came to see the second law as a statistical law, a law that characterizes a closed system of colliding gas molecules. The second law was not a universal. Nevertheless, there has been an over-extension of this statistical law masquerading as an explanation, when the second law remains starkly unexplained. Price (1996, chapter 2) notes that if the second law predicts the future elevation of entropy, then it also ridiculously predicts the elevation of past entropy. Albert (2000, chapter 4) indicates that the second law is incomplete, and what is also needed is a "past hypothesis" that stipulates a low entropy birth of the universe. The low entropy birth remains unexplained. Penrose (2004, section 27.13) relates this specialness to a big bang, which came from a repulsive gravity that was somehow transformed into an attraction. And if we are going to give meaning to the second law we must assume a closed system from the start, an ensemble of mindless molecules that collide and migrate randomly. Alternatively, attempts have been made to make sense out of the second law in an open universe (e.g., Chaisson, 2001).

For Hegel's Notion to fail under the second law, the second law must also survive as an explanation, and it has not. The negative argument is that the second law remains as unexplained as Hegel's Notion. Indeed, the second law can be recast as Hegel's second negation that negates abstract symmetry. But for this positive argument to succeed, the second law must be consistent with the intuitions of one-pointedness and route-invariance. Smith (2007) describes the second law as a duality that hides the act of self-recognition behind the act of representation. Albert (2000, chapter 7) relates thermodynamic irreversibility to the quantum mechanical collapse of the wave function. What is represented as the collapsed wave function, as much as what is represented to the mind, is indeed irreversible. And what is irreversible is therefore one-pointed.

To appreciate route-invariance one must consider the closed and gas-filled ensemble again. The heat and pressure of this container is all that is known. The particular paths taken by the colliding gas molecules are not known. Helrich (2007) tells us that “particle trajectories must be giving up”. What is expressed outwardly is only the collective behavior of mindless motion absent any collective properties. Helrich tells us that there is no present formulation for a variation principle suitable for the second law as found with laws derived from symmetries, and that such a variation principle is sought if only because it comes with a possible teleology. However, it is enough to know that what is assumed to be collective and mindless is only caricature derived from the intuition of route-invariance inflicted on a closed system. The mindless variations of the caricature are well downstream from the synthesis offered from the sensation of heat and pressure, and this is well out of reach of any teleology.

The heat death is what is expected from a mindless mass, and we would expect no less from Hegel’s second negation as the Spirit returns to its source. The second law remains bluntly real, unexplained by statistics unless we are already describing a mindless mass.

## 5. Conclusion

Space-time geometry does not have a license to caricature beyond the congruencies that are found necessary for language; these congruencies are starkly discovered and are synthetic, and it is they that provide the grounding of geometry as language. Intuition frees us from literalism because the synthetics are experiential and indicate something beyond the formality.

I have described space-time as an emotive field, a substance that provides completeness merely by showing the incompleteness in the formality offered by language. In my book, *Trinity*, I offered the completion of general relativity and quantum mechanics. I expressed the same emotion above. Chiao (2003) describes this tension as a conflict between spatial non-separability of quantum mechanics against the complete spatial separability offered by general relativity. But in describing general relativity in the language of geometry I have noted that it is only the pure formality that offers complete spatial separability. The feeling finds itself escaping the formality. The space-time points that Greene (1999, chapter 5) concerns himself with only exist in the abstract formality. The size-less point is only post-synthetic, and downstream from the congruencies that geometry declares. General relativity says nothing about the middle term that holds the congruencies together; and general relativity says nothing about the feeling that is communicated with the geometrical language. The conflict only exists within the formality, but the congruencies already hint that space-time is full of singularities that cannot be mapped.

In reducing geometry into language we discover that the conflict between general relativity and quantum mechanics does not exist beyond Hegel’s Notion. It is important to note the limits of this unification. This unification is not the same as an explanation that merely emerges from additional formality (e.g., string theory), this unification is metaphysical and avoids the infinite regress. The need for additional answers returns us to Hegel’s Notion, otherwise science need not stop in its search for a formal synthesis that explains the mystery of quantum gravity. Moreover, alternative approaches (e.g., Markopoulou 2000) may offer intuitionist accounts that better describe this tension, while respecting a truth that is necessarily dependent on the experiential; like a good story that is repeated in different words.

Formality finds itself fighting intuition, even attempting to extinguish all intuitions by reducing everything into an irritating formality. Brouwer's friendship with Hilbert was destroyed by this tension. Walter P. Van Stigt (see Mancosu 998, page 3) writes of "the unjustified and illegal dismissal of Brouwer from the editorial board of the *Mathematische Annalen* by Hilbert in 1928." Like the middle term, Brouwer found himself excluded because he expressed his emotions. However, the drive to formality meets its demise in Hegel's second negation. This is the point where euphoria is resurrected.

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